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Variable Digital Filter Response Time in a Digital Distance Relay

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INTRODUCTION

This paper will investigate the effect of digital filtering on the operating time of a generic digital distance relay. Two popular filtering approaches will be considered: the Cosine Filter and the Fourier Filter. The effects of several variables, such as sampling rate, fault location, fault incidence angle, and non-fundamental frequencies, will be discussed. This paper will attempt to discuss the performance of the digital filters in terms familiar to most protection engineers.

DISTANCE RELAY OPERATING PRINCIPLES

A previous discussion of digital distance relay filtering [3] used the voltage divided by the current as an indication of the performance of a distance relay. While this method may help in the understanding of how the impedance is "seen" by a distance relay, it does not accurately predict the performance of a distance relay as most available distance relays do not operate on this principle. In order to provide a better model, a simple phase angle comparator mho distance relay will be considered in this paper. Many commercially available distance relays operate on a variation of this principle. This approach to distance relays has been well documented in the past [1, 2]. The relay chosen is a very rudimentary approximation of a true distance unit for ease of analysis. The simplifications do not affect the ideas that will be presented. The operating quantity is IZR - V; where ZR is the set relay reach and V and I are the voltage and current signals supplied to the relay. The polarizing voltage is assumed to follow the pre-fault voltage; this approximates the many variations of polarizing and memory voltages used in distance relays considering the short duration of interest. Operation of the distance element occurs when the phase angle between the operating and polarizing signals is less than ±90 electrical degrees.

DIGITAL FILTERING

The phase angle comparator method of implementing a distance relay is based on the fundamental frequency components in the currents and voltages supplied to the relay. In a typical analog relay, bandpass filters tuned to the fundamental frequency are used to eliminate unwanted frequencies, both higher and lower than the fundamental. A digital relay will include analog filtering, but it is intended for anti-aliasing purposes rather than the removal of non-fundamental frequency components [4]. The anti-aliasing filter is a low pass filter with a maximum cutoff frequency determined by the sampling rate of the relay.

The phase angle comparator principle uses the phasor information contained in the input signals. A digital filter that both removes the non-fundamental frequencies and also provides phasor information is desirable for a digital implementation of a phase angle comparator distance relay. Two such filters will be considered in this paper: the Discrete Fourier Transform and the Cosine Filter.

A steady state voltage signal in the time domain can be described by the equation:

$$v(t) = V_{peak} \bullet \sin(\omega t + \theta)$$

In a digital relay, this signal is sampled N times per cycle. Thus the input is represented by a series of samples, S_k, where k = 1 to N.

Digital filters, such as those discussed in this paper, process the sampled data points, S_k, by multiplying each sample by a coefficient determined by the type of digital filter employed. This process is described in the following sections.

Recursive Vs Non-recursive Filters

There are two methods of calculating the Discrete Fourier Transform: Recursive and Non-recursive. The Non-recursive method requires that each sampled data point be saved in memory (amount of data is determined by the "window" size) and that the entire summation process be performed for every sample. The oldest sample becomes the initial sample and the newest sample becomes the Nth sample. The real and imaginary terms must be recalculated from the beginning. The Recursive method requires that the product of the sine and cosine coefficients and the sample data values used to generate the sums are saved (the amount is still determined by the "window" size) and an abbreviated summation process is performed. In this method, the oldest product is removed from the sum and the newest product is added into the sum. Now, only the values for the latest sample need to calculated instead of having to calculate the values for all the samples in the "window". This reduces the amount of calculations performed. Therefore, the time required to complete this process is also reduced enabling the relay to perform additional tasks or increase its sampling rate. The Recursive method, on the other hand, requires more time and/or computing speed to complete.

Discrete Fourier Transform

Before getting into the details of the Fourier Transform Calculations, the phasor definitions should first be described. The following equation is for a sinusoidal voltage. There is a similar one for a sinusoidal current but only the voltage representation is given here as an example.

$$v(t) = V_{peak} \bullet \sin(\omega t + \theta_V)$$

Expanding the above equation:

$$v(t) = V_{peak} \bullet \cos(\theta_V) \bullet \sin(\omega t) + V_{peak} \bullet \sin(\theta_V) \bullet \cos(\omega t)$$

When v(t) is sampled, the resulting sample values are denoted by S_k. Since S_k represents sampled values with a fixed sample rate of N samples per cycle of sinusoidal voltage, the Discrete Fourier Transform Calculation of the fundamental components can be defined by the following equations.

$$Vreal = \left(\frac{2}{N}\right) \bullet \sum_{k=1}^{N} [Sk \bullet \sin(2 \bullet \pi \bullet \frac{k}{N})]$$

$$Vimag = \left(\frac{2}{N}\right) \bullet \sum_{k=1}^{N} [Sk \bullet \cos(2 \bullet \pi \bullet \frac{k}{N})]$$

Applying these equations to the original voltage equation results in the following expressions.

$$Vreal = Vpeak \bullet \cos(\theta_V)$$

$$Vimag = Vpeak \bullet \sin(\theta_V)$$

The magnitude of the voltage phasor can be calculated by the following equation.

$$Vmag = (Vreal^2 + Vimag^2)^{½}$$

$$Vmag = (Vpeak^2 \bullet (\cos(\theta_V)^2 + \cos(\theta_V)^2)^{½}$$

$$Vmag = Vpeak$$

The phase angle of the voltage phasor can be calculated by the following equation. Note that for a Recursive filter, the angle θ_V is constant, while for a Non-recursive filter it is rotating .

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$$Vangle = \arctan(Vimag/Vreal)$$

$$Vangle = \theta_V$$

With these definitions, the Fourier Transform Calculation is able to convert the sinusoidal voltage waveshape to a phasor. The phasor is represented by two forms, the first form is the rectangular form where the real and imaginary components define the phasor; the second form is the polar form where the magnitude and the phase angle define the phasor.

The Fourier Transform Calculation determines the real and imaginary parts of each of the currents and voltages used in the relay. This means that each current and voltage sample is multiplied by a sine factor to obtain the real component and by a cosine factor to obtain the imaginary component. These quantities will then be summed over N consecutive samples to obtain the actual components.

For this paper, the Fourier Transform Calculation uses a recursive concept to perform the summation of the real and imaginary terms. This means that instead of re-calculating the sine and cosine factors and re-summing them every sample, only the sine and cosine factors for the present sample are calculated. Then the oldest sample's sine and cosine terms are removed from the sum and the newest terms are added in to the sum.

$$Vreal(k) = Vreal(k-1) + S_k \bullet \cos() - S(k-N) \bullet \cos()$$

$$Vimag(k) = Vimag(k-1) + S_k \bullet \sin() - S(k-N) \bullet \sin()$$

This requires that each sine and cosine product be save until they are removed from the sum. In addition the actual sum must also be saved since both the "k-1" sum for the previous sample and the "k" sum for the present sample are used in the recursive calculation. After the recursive calculation has been performed for each sample, the values are updated. This approach will reduce the amount of time required to perform the Fourier Transform Calculations. If an error is somehow introduced into a sum, then subtracting out the old sine and cosine factors and adding in the new ones will not remove it. Techniques are available to prevent this error from affecting the filter output.

Half Cycle Fourier

The Discrete Fourier Transform has the capability of working on different sized "windows". In the following paragraph the option of using a Half Cycle window will be discussed. The Full Cycle window generates the sums using all the sampled data collected over the last cycle. This means that the "window" includes the last full cycle's worth of data. The Half Cycle window generates the sums using the sampled data collected in the last half cycle. Therefore, the data "window" is a half cycle. Using a Half Cycle window allows the Discrete Fourier Transform to more quickly track a change in the sampled data than is possible with a Full Cycle window. Also, less data will need to be saved for Half Cycle window than a Full Cycle window. However, there are differences in the filtering actions of the Half and Full Cycle filters.

In a 16 sample Full Cycle Fourier Filter the sums are generated using the last 16 samples. In a Half Cycle Fourier Filter they would be generated using the last 8 samples. Since the amount of data within the sums is less, this has an effect on the constant that is used to multiply the sums; thus (2/N) is replaced by (4/N). The sine and cosine coefficient terms are determined by the factor $(2 \bullet \pi \bullet k/N)$. Therefore the sine and cosine coefficients used within the sums are still determined by the sampling rate rather than the filter "window" size. The Half Cycle Fourier may be implemented as either a Recursive or Non-recursive filter.

Cosine Filter

The Discrete Fourier Transform described previously uses sine and cosine coefficients to develop the real and imaginary components of the filtered signals. In order to obtain the real and imaginary components, two signals 90 electrical degrees apart are required; the sine and cosine coefficient meet this requirement in the Fourier approach. Another approach is to only use the coefficients from the cosine with the second signal being the cosine value from the calculation 90 electrical degrees previous. This method requires that only one set of calculations be performed for each sample rather than the two sets required with the Fourier. This savings, however, is at least partially offset by the fact that the cosine approach must be non-recursive while the Fourier may be a recursive filter. The Cosine filter uses a one and a quarter cycle window rather than the one cycle window used by the Fourier, and therefore slower operating times may be expected. The 4 sample per cycle Cosine filter is of particular interest as the previously calculated term is in fact the required quadrature sample.

REPLICA IMPEDANCE CIRCUITS

A previous paper [3] implies that the Fourier filter is not appropriate for use in protective relays as it does not remove the DC offset component of current. Relay design engineers have been accustomed to dealing with the effects of DC offset for many, many years. There are a wide variety of methods that can effectively remove the DC component from the current.

The majority of distance relays use an IZ signal rather than an I signal. This Z is typically referred to as the "replica impedance" or "mimic impedance". The name is derived from the fact this impedance is ideally the same as that of the protected line. This IZ term is the one commonly used to create the distance relay operating signal, IZR - V. In steady state conditions, this Z can be thought of as a phasor multiplying the current; this is the usual technique used to show the development of the mho characteristics [1,2]. However, during transient conditions, the replica impedance is more complex than a simple phasor multiplication.

Many solid state relay systems use magnetic circuits such as a transactor to develop the transmission line replica impedance. A transactor is an iron core reactor with an air gap. The transactor produces a voltage proportional to the input current. The transfer impedance of the transactor is defined as Z, and is used to determine the reach and angle of maximum reach of the mho characteristic. The transactor removes the DC offset component from the current signal. Other techniques may be used in digital relays: for example, a software implementation of the transactor to create the replica impedance. Thus the DC component can be removed from the current derived signal that is used by a digital relay.

In order to illustrate the effects of the replica impedance, consider the simple system of Figure 1. A fault is applied at 75% of the line at zero degrees on the voltage wave. The resulting current is shown in Figure 2. This current was processed by a 16 sample per cycle Full Cycle Fourier Filter. The magnitude of the Fourier output is plotted in Figure 3; the phase angle is plotted in Figure 4. It can be seen in these plots that both the magnitude and angle of the phasor obtained from the full cycle Fourier oscillates, and does not approach its steady state value for approximately 4 - 5 cycles. The maximum error after the sixteenth sample is approximately ten percent. The same sampled current was passed through a digital implementation of a replica impedance with a unity magnitude and a phase angle of 85 degrees to match the line angle. The results are shown in Figures 5 and 6. In this case, the output of the Fourier filter reaches its steady state value in approximately 1 cycle, without the oscillations that were evident in the current signal. The real and imaginary parts of the current and IZ are plotted in Figures 7 and 8, respectively. The same current waveform was also processed by a Cosine Filter. Figures 9 and 10 show the magnitude of the Current (I) and IZ signals. The digital model of the transactor has had a smoothing effect on the output of the Cosine Filter as well.

OPERATING TIMES

There are several factors in a digital relay that will affect the operating time of the relay when a fault occurs. One of these is the time delay associated with the anti-aliasing filter. All analog filtering introduces a time delay. This time delay is dependent upon the type of filter (low pass, bandpass, high pass), the "cutoff" frequency of the filter, and the "rolloff" characteristic. The time delays associated with the anti-aliasing filter are ignored in this paper.

A second source of time delay is due to the nature of sampled data. An analog relay begins to respond to a change in current or voltage the instant it occurs; a digital relay, on the other hand, must wait until the next time that the input signals are sampled. When the fault occurs just after a sample, the relay must wait one full sample time before it can sense a change in the inputs; for a relay that samples 32 times per 60 Hz cycle, this is 0.52 ms, while for a relay that samples 4 times per cycle, it is 4.17 ms.

A third factor that will have a direct effect on the relay operating time is the "security count" used in the protection algorithm. The security count refers to the number of samples that must be within the "tripping zone" of the relay before an actual trip output is permitted. A digital relay will not normally produce a trip output as the result of a single calculation; this is intended to prevent a misoperation if the trip calculation is the result of some error rather than actual system conditions. The higher the security count, the longer the relay must wait to issue a trip command. For the purposes of this paper, a security count of 2 samples will be used. A 2 sample count at 32 samples per cycle will delay tripping 0.52 ms; while at a sample rate of 4 per cycle the delay will be 4.17 ms.

Another factor that affects the operating time is the time delay introduced by the digital filtering. It is the variations in this time based on the severity of the fault that will now be addressed. Faults were applied at 0, 25 50 and 75 percent of the radial line of Figure 1. Sampled currents and voltages were calculated and processed by the various digital filters. For simplicity and consistency, the fault occurs simultaneously with sample number 1, regardless of the sampling rate chosen. The outputs of the filters were used to form the distance relay operating signal, IZ-V. The phase angle of IZ-V signal was compared to the phase angle of the prefault memory voltage; when the two were within ±90°, a trip condition was detected. Two consecutive trip conditions were required to issue a trip signal.

The Tables on the following pages summarize the operating times for the various filter and sampling rate combinations. As might be expected, the fastest times for a given filter are produced with the highest sampling rate. The operating times for the close-in faults are of particular interest. Both the Full Cycle Fourier and Cosine filters produce a trip output, including a two count security delay, in 4.7 ms, with a sampling rate of 32 samples per cycle; however, when the sampling rate is reduced to 16 ,8 or 4, the trip times for the Cosine filter are one quarter cycle longer than for the Fourier.

The fault initiation angle was shifted 90 degrees for a fault at 75% of the line. A comparison of the trip times is shown below:

Fault Initiation Angle	16 Sample Full Cycle Fourier Trip Sample	16 Sample Cosine Trip Sample
0	15	19
90	20	20

For this fault, the Fourier trip time was faster than the Cosine for an initiation angle of 0 degree, but the trip times were the same for an initiation angle of 90 degrees.

Figure 11 shows the Comparator Angle (angle between IZR - V and Vpol) for the 16 sample Full Cycle Fourier with a fault at 75% and a 0 degree fault incidence angle. Figure 12 shows the same information for the 16 sample Cosine filter. Figures 13 and 14 show the Comparator Angle for a 90 degree incidence angle.

All of the previous data was based on the system of Figure 1, which has a source to line impedance ratio (SIR) of 0.25. The source impedance was increased to 40 ohms, to produce a SIR of 10. For a fault at the relay location (F1), the 32 sample per cycle Fourier Filter produces a trip output in 14.6 ms; the 32 sample per cycle Cosine filter trips in 18.7 ms. For an SIR of 0.25, both filters produced trip outputs in 4.7 ms. Thus the operating time will be a function of the SIR as well as the fault location on the line.

SUMMARY

It is a common assumption that if a full cycle digital filter is used, that a one cycle delay can be expected in the operating times. As shown in this discussion, however, this is not always true. A phase angle comparator distance relay, for example, does not measure the exact distance to the fault; rather, it produces a trip output whenever the calculated phase angles are within its tripping zone. Thus, while it may take one cycle for the fault impedance to reach ±1% of its final value, the impedance may enter its tripping characteristic within several samples. This is especially true for faults well within the set relay reach. Figure 15 is a plot of the tripping times for a phase angle comparator relay using Full Cycle Fourier filter as a function of the sample rate. The slowest trip time for this condition was on half cycle. Figure 16 summarizes the data of Table I and shows that the relaying tripping times increase as the fault location moves down the line. This type of performance is similar to that of many analog distance relays.

REFERENCES

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[2] G.E. Alexander and J.G. Adrichak, "Ground Distance Relaying: Problems and Principles", Western Protective Relay Conference, 1992

[3] Daqing Hou and E.O. Schweitzer, "Filtering for Protective Relays", Western Protective Relay Conference, 1992

[4] Arun Phadke and James S. Thorp, "Computer Relaying for Power Systems", John Wiley and Sons Inc., 1988

TABLE I
FULL CYCLE FOURIER

Fault at 0 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	2	3	8.3
8	3	4	6.2
16	5	6	5.2
32	9	10	4.7

Fault at 25 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	3	4	12.5
8	4	5	8.3
16	7	8	7.3
32	12	13	6.2

Fault at 50 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	2	3	8.3
8	3	4	6.2
16	5	6	5.2
32	9	10	4.7

Fault at 75 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	3	4	12.5
8	4	5	8.3
16	7	8	7.3
32	12	13	6.2

TABLE II
HALF CYCLE FOURIER

Fault at 0 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	2	3	8.3
8	3	4	6.2
16	4	5	4.2
32	7	8	3.6

Fault at 25 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	2	3	8.3
8	3	4	6.2
16	5	6	5.2
32	9	10	4.7

Fault at 50 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	3	4	12.5
8	4	5	8.3
16	6	7	6.3
32	10	11	5.2

Fault at 75 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	3	4	12.5
8	4	5	8.3
16	7	8	7.3
32	12	13	6.2

TABLE III
FULL CYCLE COSINE

Fault at 0 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	3	4	12.5
8	5	6	10.4
16	9	10	9.4
32	9	10	4.7

Fault at 25 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	3	4	12.5
8	6	7	12.5
16	11	12	11.5
32	20	21	10.4

Fault at 50 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	5	6	20.8
8	9	10	18.7
16	12	13	12.5
32	23	24	12.0

Fault at 75 % of Line

Sample Rate	First Trip Condition From Filter (Sample)	Relay Trip Output Sample	Relay Trip Output Milliseconds
4	5	6	20.8
8	10	11	20.8
16	18	19	18.7
32	35	36	18.2

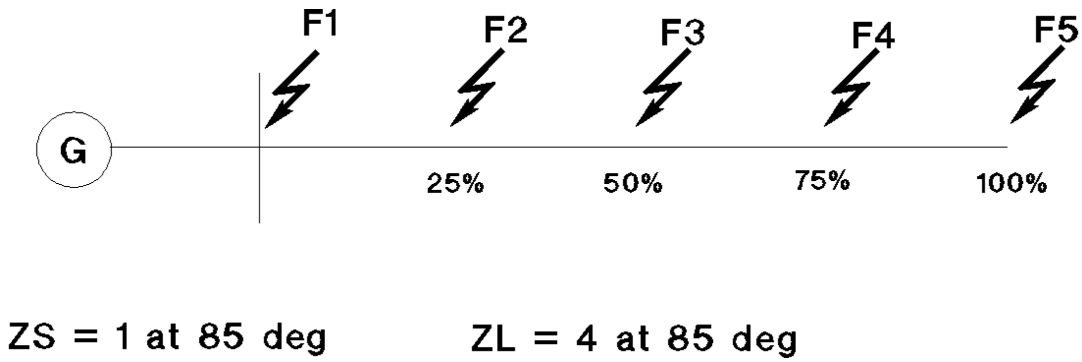
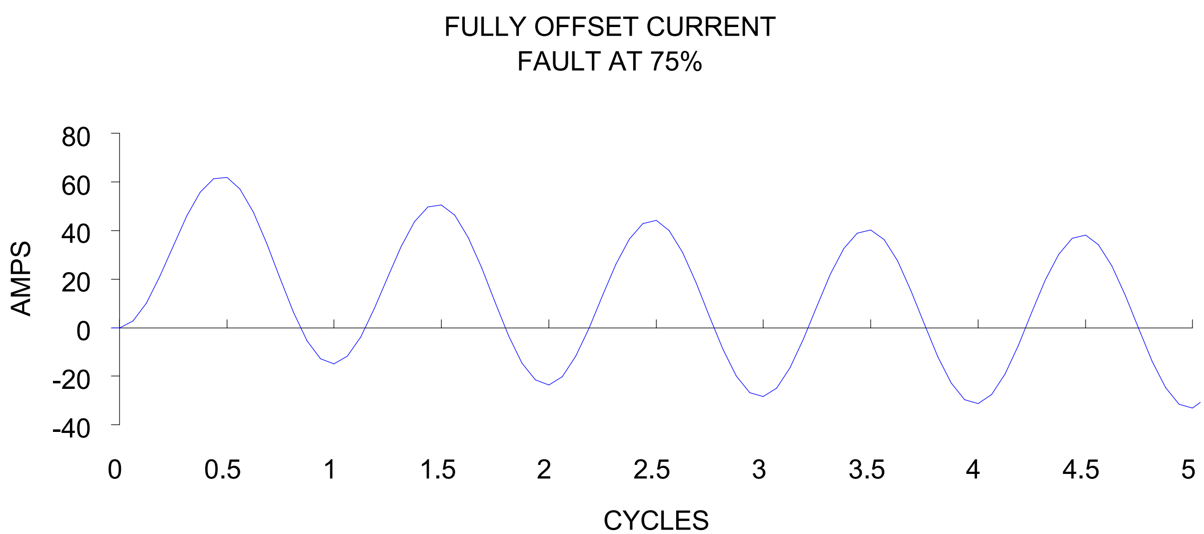


Figure 1



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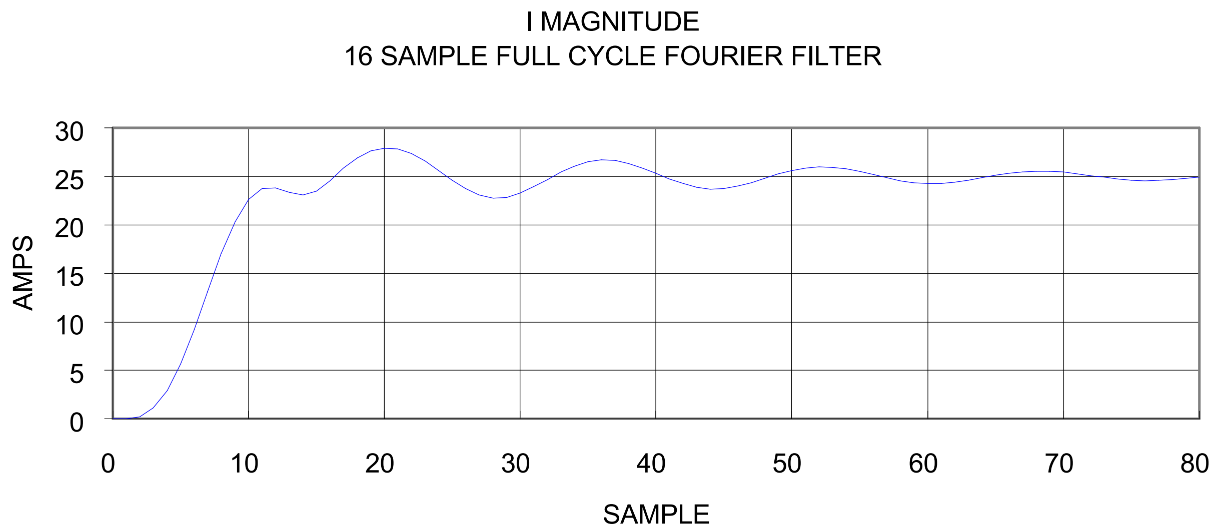

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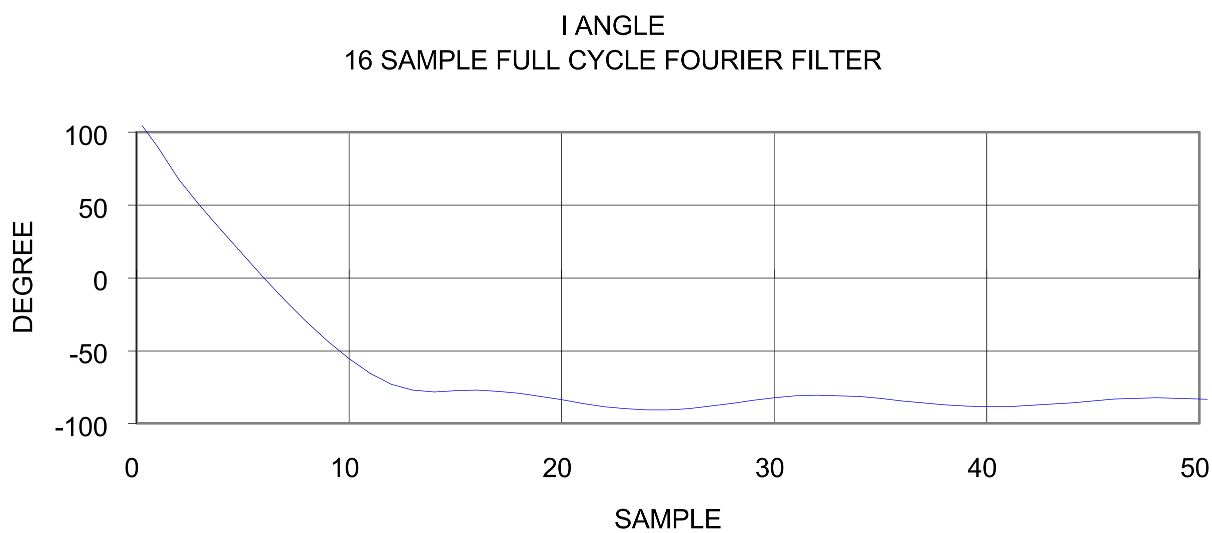

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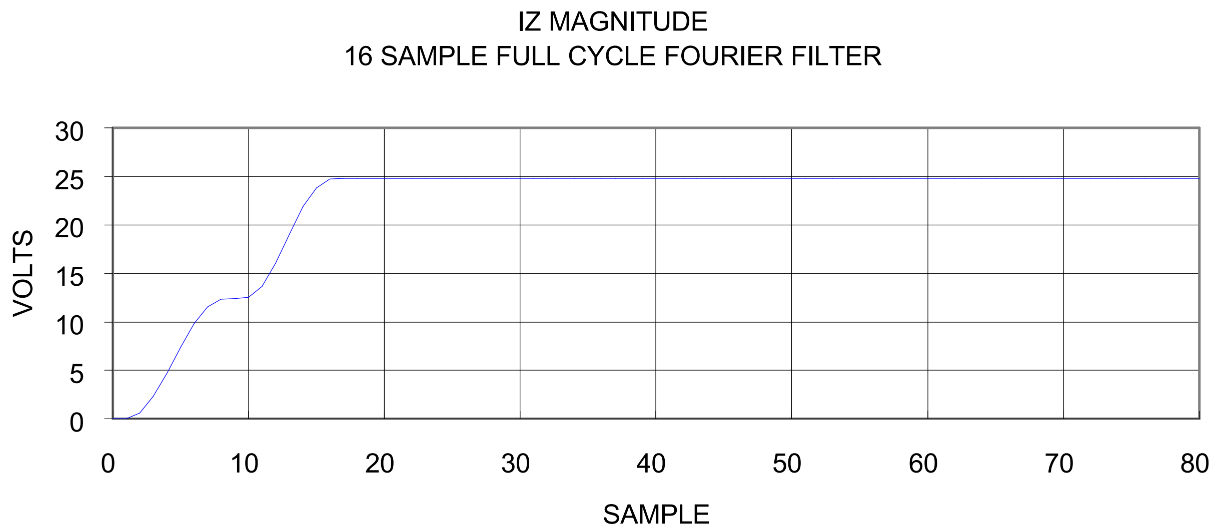

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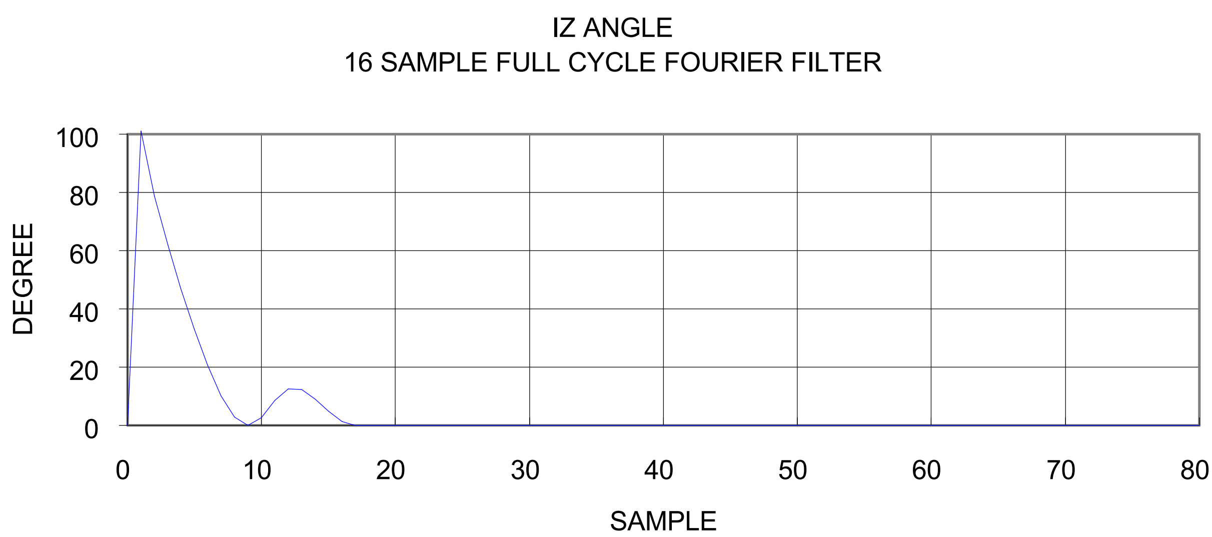

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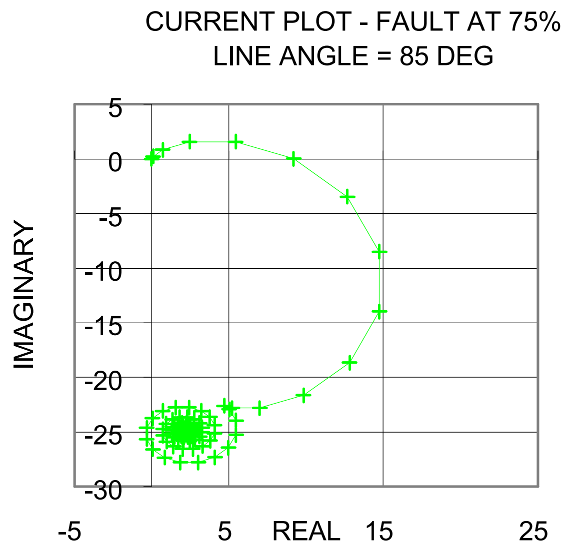

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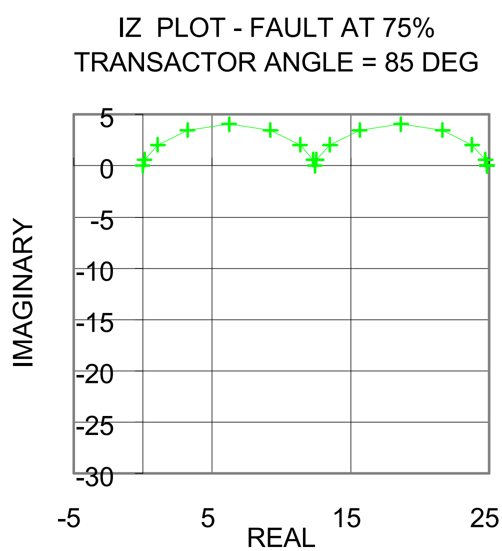

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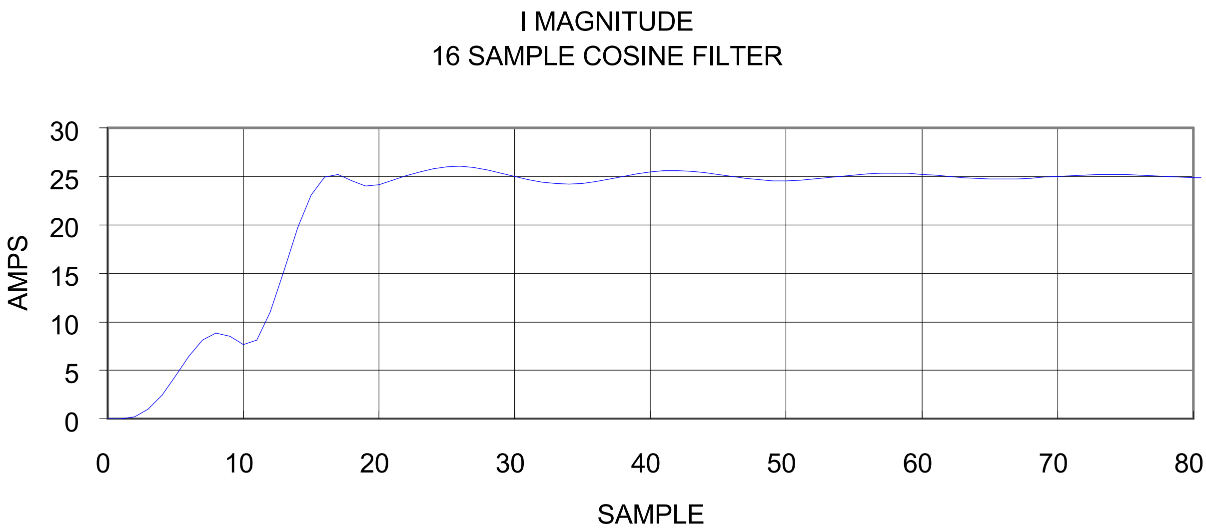

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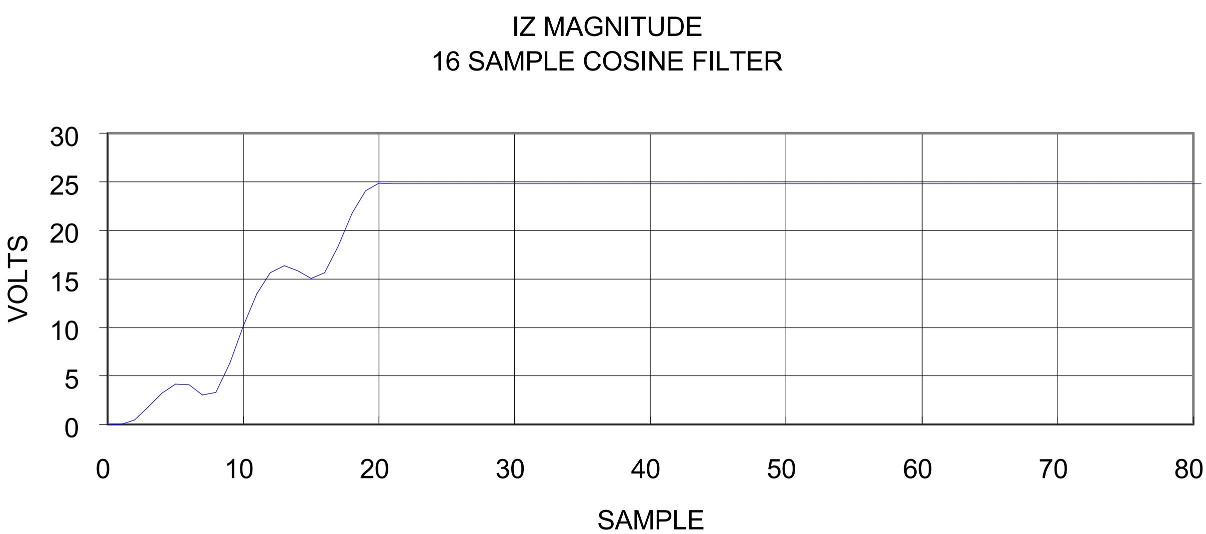


Figure 10

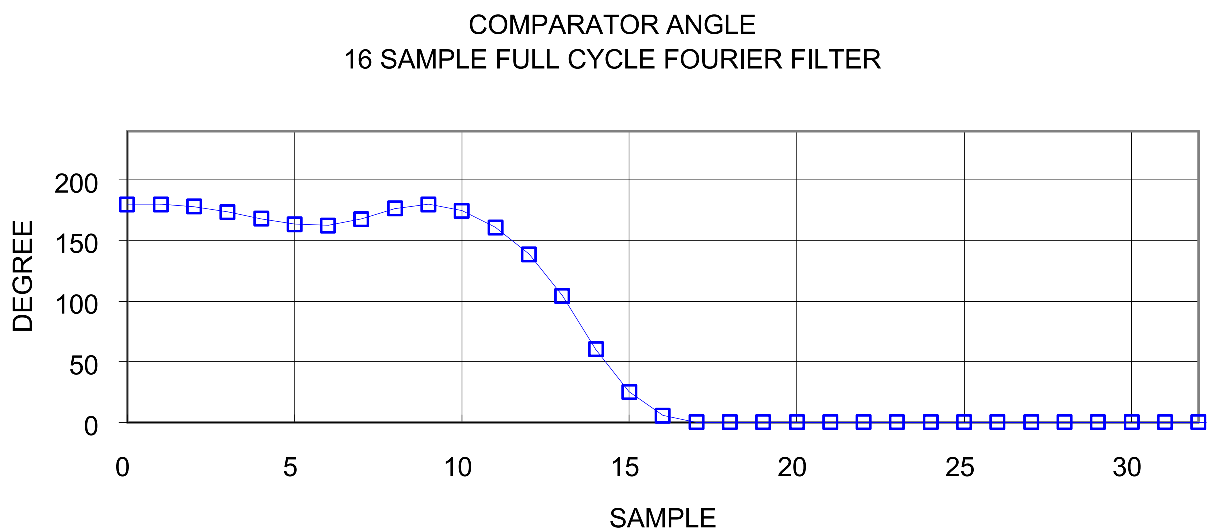
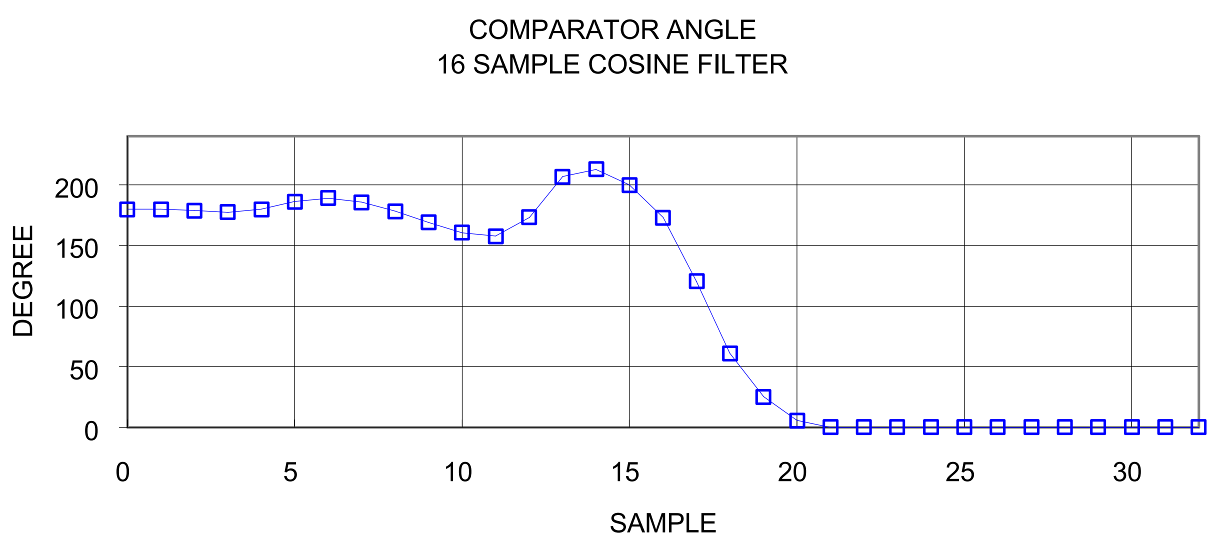


Figure 11



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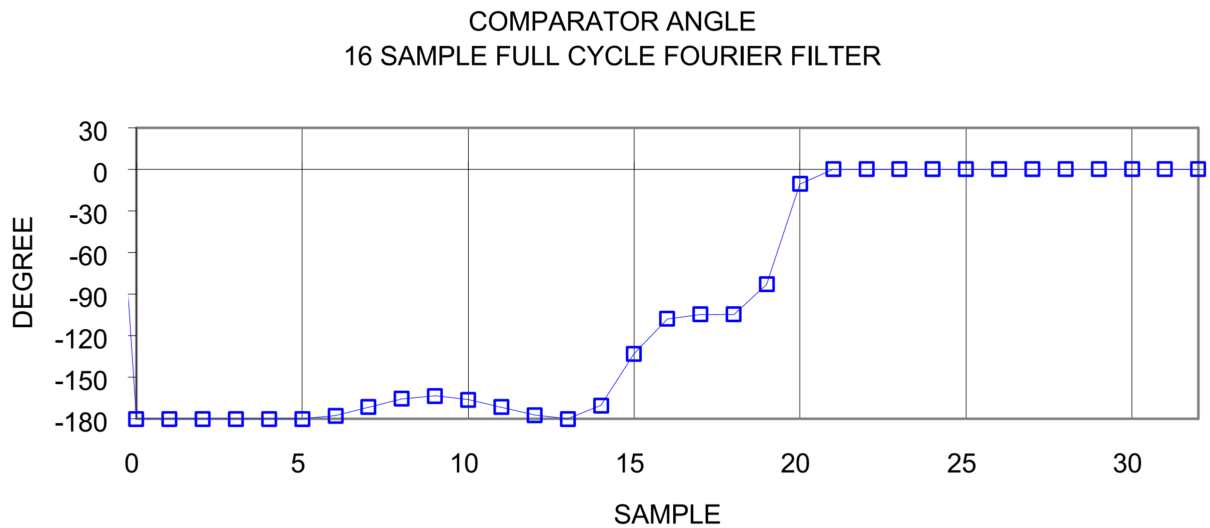

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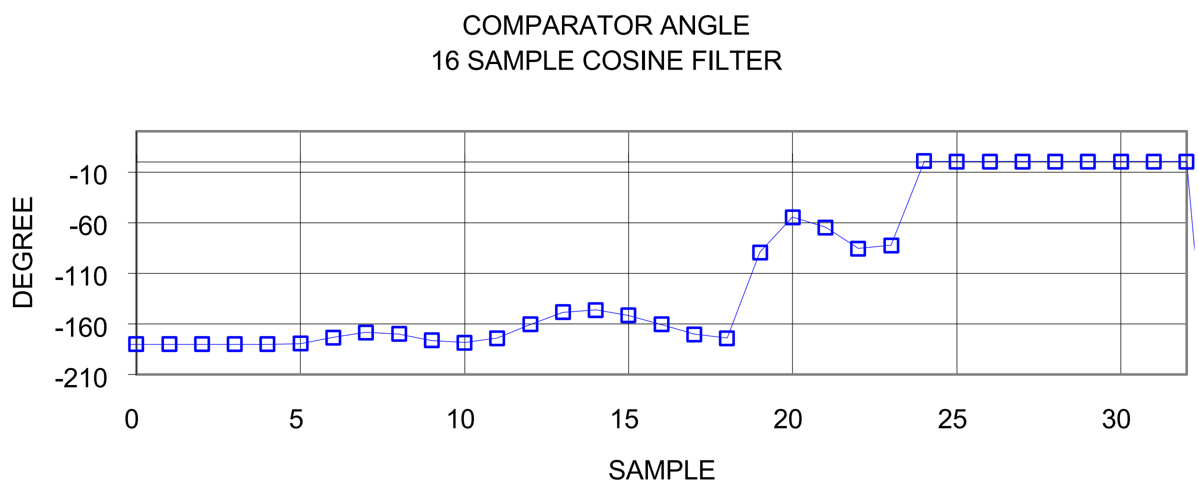

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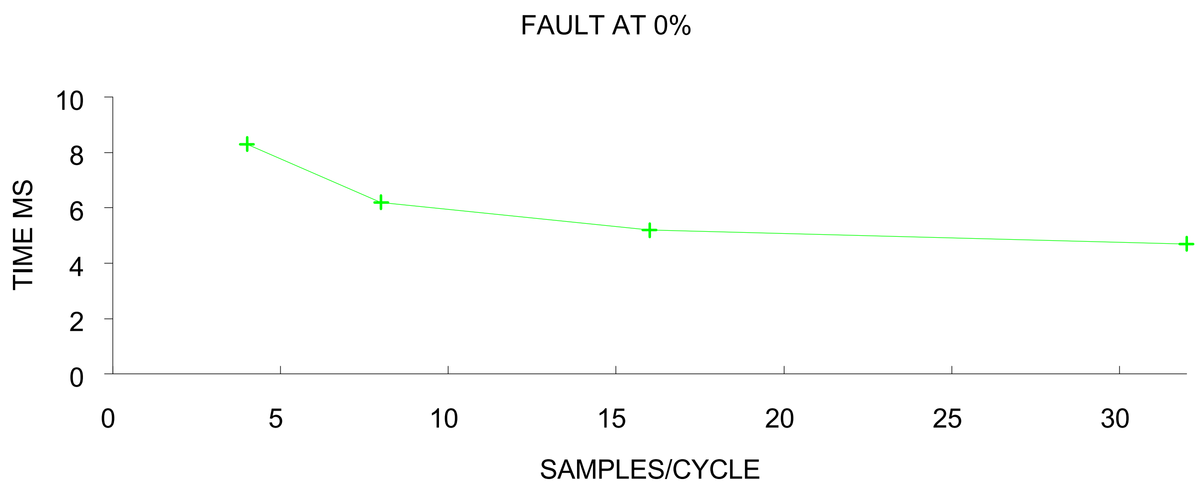

Figure 15

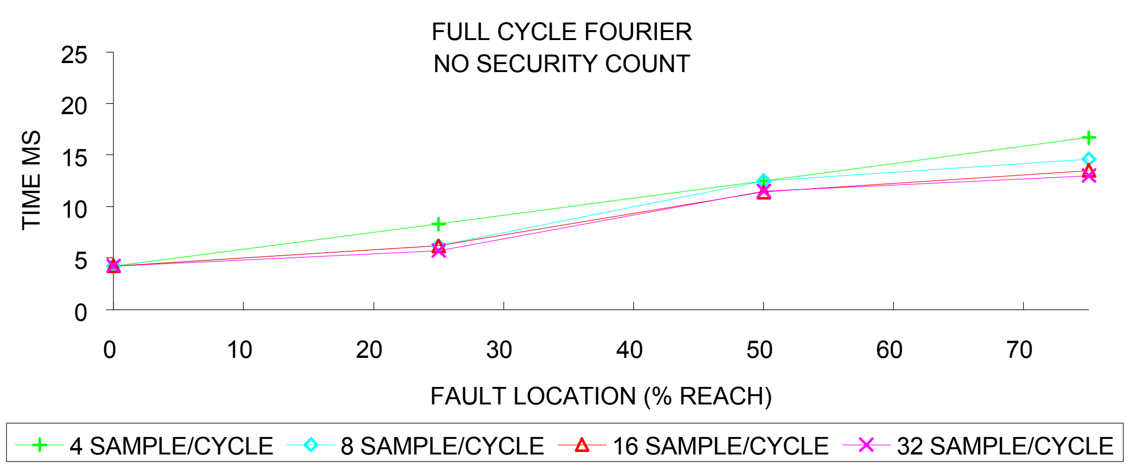

Figure 16