



# An Improved Transformer Inrush Restraint Algorithm

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**AN IMPROVED TRANSFORMER INRUSH RESTRAINT  
ALGORITHM INCREASES SECURITY WHILE MAINTAINING  
FAULT RESPONSE PERFORMANCE**

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### *Abstract*

This paper presents a new inrush restraint algorithm for the protection of power transformers. The algorithm is an extension of the traditional second harmonic restraint — instead of measuring the ratio between the magnitudes of the second harmonic and the fundamental frequency component, the algorithm considers a ratio between the phasors of the second and the fundamental frequency components of the differential signal, i.e. both the amplitude and phase relations. By making use of the additional dimension that was neglected until now, the new algorithm is capable of making more robust classification of differential currents caused by the magnetizing inrush phenomenon and those caused by true internal faults. The algorithm is presented in detail. Its analytical justification is supported by the results of numerical analysis, including digital simulation and waveforms recorded from physical transformers.

## **1. Introduction**

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Large power transformers belong to a class of vital and very expensive components in electric power systems. If a power transformer experiences a fault, it is necessary to take the transformer out of service as soon as possible so that the damage is minimized. The costs associated with repairing a damaged transformer may be very high. The unplanned outage of a power transformer can also cost electric utilities millions of dollars. Consequently, it is of a great importance to minimize the frequency and duration of unwanted outages. Accordingly, high demands are imposed on power transformer protective relays. The requirements include dependability (no missing operations), security (no false trippings), and speed of operation (short fault clearing time).

The operating conditions of power transformers, however, do not make the relaying task easy. Protection of large power transformers is perhaps the most challenging problem in the area of power system relaying.

Table 1 reviews the basic problems of transformer differential relaying from the perspective of magnetizing inrush, stationary overexcitation of a core, internal and external faults, all in the context of measurements, security, dependability and speed of operation [1,2].

Numerical relays capable of performing sophisticated signal processing enable the relay designer to re-visit the classical protection principles and enhance the relay performance, facilitating faster, more secure and dependable protection for power transformers [3].

This paper addresses the issue of restraining a transformer differential relay during magnetizing inrush conditions.

Table 1. Problems Related To Protective Relaying Of Power Transformers

Disturbance	Measurement	Security	Dependability	Speed
Inrush	Accurate estimation of the 2nd and the 5th harmonics takes around one cycle. Off-nominal frequencies create extra measuring errors in harmonic ratio estimation	In modern power transformers, due to the magnetic properties of the core, the 2nd harmonic during inrush and the 5th harmonic during overexcitation may be very low jeopardizing relay security	The presence of higher harmonics does not indicate necessarily inrush. The harmonics may block a relay during severe internal faults due to saturation of the CTs	It usually takes one full cycle to reject the magnetizing inrush and stationary overexcitation hypotheses if an internal fault is not severe enough to be tripped by the unrestrained differential element
Overexcitation			The 5th harmonic may be present in internal fault currents due to saturation of the CTs, and due to rotor asymmetry of generators and/or power electronic devices	
External faults	The measured currents display enormous rate of change and are often significantly distorted	External fault current when combined with ratio mismatch may generate a false differential signal. The CTs, when saturated during external faults, may produce an extra differential signal	All the means of preventing false tripping during external faults reduce to a certain extent the dependability of the relay	The means of restraining the relay from tripping during external faults may limit the relay speed of operation
Internal faults		The internal fault current may be as low as few percent of the rated value. Attempts to cover such faults jeopardize relay security	The internal fault current may be as low as a few percent of the rated value. The security demands under inrush, overexcitation and external faults may limit relay dependability	The means of restraining the relay from tripping during inrush, overexcitation and external faults may limit the relay speed of operation

## 2. Magnetizing Inrush — A Brief Analysis

Magnetizing inrush current in transformers results from any abrupt change of the magnetizing voltage. Although usually considered a result of energizing a transformer, the magnetizing inrush may be also caused by [4,5]:

- (a) occurrence of an external fault,
- (b) voltage recovery after clearing an external fault,
- (c) change of the character of a fault (for example when a phase-to-ground fault evolves into a phase-to-phase-to-ground fault), and
- (d) out-of-phase synchronizing of a connected generator.

Since the magnetizing branch representing the core appears as a shunt element in the transformer equivalent circuit, the magnetizing current upsets the balance between the currents at the transformer terminals, and is therefore experienced by the differential relay as a “false” differential current. The relay, however, must remain stable during inrush

conditions. In addition, from the standpoint of the transformer life-time, tripping-out during inrush conditions is a very undesirable situation (breaking a current of a pure inductive nature generates high overvoltage that may jeopardize the insulation of a transformer and be an indirect cause of an internal fault).

### **2.1. Inrush due to switching-in**

Initial magnetizing due to switching a transformer in is considered the most severe case of an inrush. When a transformer is de-energized (switched-off), the magnetizing voltage is taken away, the magnetizing current goes to zero while the flux follows the hysteresis loop of the core. This results in certain remanent flux left in the core. When, afterwards, the transformer is re-energized by an alternating sinusoidal voltage, the flux becomes also sinusoidal but biased by the remanence. The residual flux may be as high as 80-90% of the rated flux, and therefore, it may shift the flux-current trajectories far above the knee-point of the characteristic resulting in both large peak values and heavy distortions of the magnetizing current (Figure 1).

Figure 2 shows a typical inrush current. The waveform displays a large and long lasting dc component, is rich in harmonics, assumes large peak values at the beginning (up to 30 times the rated value), decays substantially after a few tenths of a second, but its full decay occurs only after several seconds (to the normal excitation level of 1-2% of the rated current). In certain circumstances, some small changes of the excitation current are observable even minutes after switching a transformer in [4,5].

The shape, magnitude and duration of the inrush current depend on several factors.

#### *A. Size of a transformer*

The peak values of the magnetizing inrush current are higher for smaller transformers while the duration of this current is longer for larger transformers. The time constant for the decaying current is in the range of 0.1 of a second for small transformers (100kVA and below) and in the range of 1 second for larger units.

#### *B. Impedance of the system from which a transformer is energized*

The inrush current is higher when the transformer is energized from a powerful system. Moreover, the total resistance seen from the equivalent source to the magnetizing branch contributes to the damping of the current. Therefore, transformers located closer to the generating plants display inrush currents lasting much longer than transformers installed electrically away from the generators.

C. Magnetic properties of the core material

The magnetizing inrush is more severe when the saturation flux density of the core is low. Designers usually work with flux densities of 1.5 to 1.75 tesla. Transformers operating closer to the latter value display lower inrush currents [4,5].

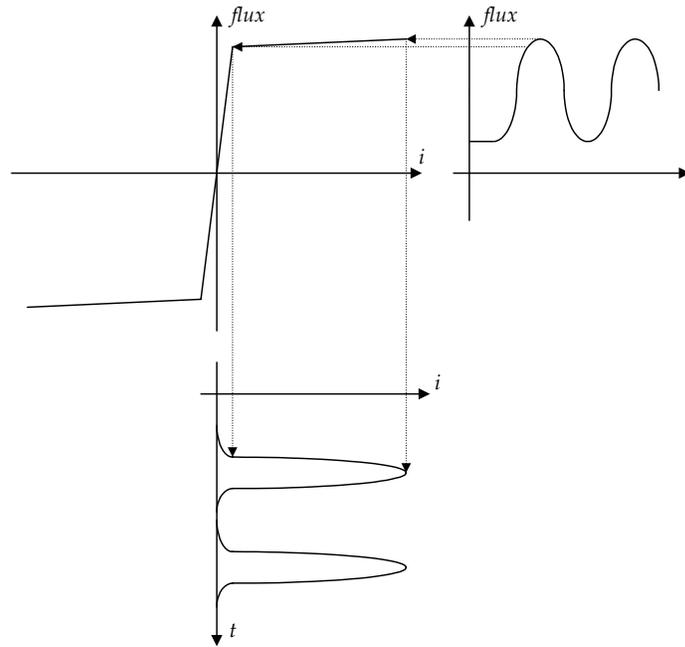


Figure 1. Illustration of the magnetizing inrush.

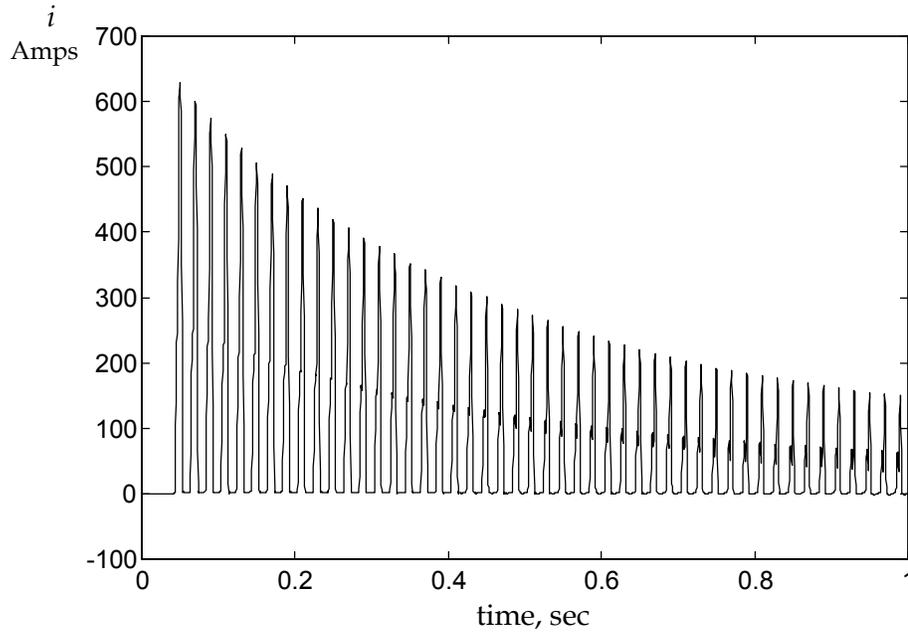


Figure 2. Typical inrush current.

#### *D. Remanence in the core*

Under the most unfavorable combination of the voltage phase and the sign of the remanent flux shown in Figure 1, higher remanent flux results in higher inrush currents. The residual flux densities may be as high as 1.3 to 1.7 tesla [4,5].

#### *E. Moment when a transformer is switched in*

The highest values of the magnetizing current occur when the transformer is switched at the zero transition of the winding voltage, and when in addition, the new forced flux assumes the same direction as the flux left in the core (Figure 1). In general, however, the magnitude of the inrush current is a random factor and depends on the point of the voltage waveform at which the switchgear closes, as well as on the sign and value of the residual flux. It is approximated that every 5th or 6th energizing of a power transformer results in considerably high values of the inrush current [6].

#### *F. Way a transformer is switched in*

The maximum inrush current is influenced by the cross-sectional area between the core and the winding which is energized. Higher values of the inrush current are observed when the inner (having smaller diameter) winding is energized first. It is approximated, that for transformers with oriented core steel, the inrush current may reach 5-10 times the rated value when the outer winding is switched-in first, and 10-20 times the rated value when the inner winding is energized first. Due to the insulation considerations, the lower voltage winding is usually wound closer to the core, and therefore, energizing of the lower voltage winding generates higher inrush currents.

Some transformers may be equipped with a special switchgear which allows switching-in via a certain resistance [4,5]. The resistance reduces the magnitude of inrush currents and substantially increases their damping. In such a case, the operating requirements for the differential protection are much more relaxed.

In contrast, when a transformer is equipped with an air-type switch, then arcing of the switch may result in successive half cycles of the magnetizing voltage of the same polarity. The consecutive same polarity peaks cumulate the residual flux and reflect in a more and more severe inrush current. This creates extreme conditions for transformer protection and jeopardizes the transformer itself [2,4,5].

## **2.2. Harmonic content of the inrush current**

Let us assume the analytical approximation shown in Figure 3 for calculation of the frequency spectrum of the inrush current. The angle  $\alpha$  is assumed to be a parameter. The amplitude of the  $n$ -th harmonic of the waveform of Figure 3 is calculated as:

$$A_n = \frac{I_m}{\pi} \left[ \frac{1}{n+1} \sin((n+1)\alpha) + \frac{1}{n-1} \sin((n-1)\alpha) - 2 \cos\left(\frac{\alpha}{n}\right) \sin(n\alpha) \right] \quad (1)$$

Figure 4 presents the frequency spectrum of the signal shown in Figure 3 calculated with the use of (1) for  $\alpha = 60, 90$  and  $120$  degrees, respectively. As seen from the figure, the second harmonic always dominates because of a large dc component. However, the amount of the second harmonic may drop below 20%. The minimum content of the second harmonic depends mainly on the knee-point of the magnetizing characteristic of the core. The lower the saturation flux density, the higher the amount of the second harmonic. Modern transformers built with improved magnetic materials have high knee-points, and therefore, their inrush currents display a comparatively low amount of the second harmonic. Since the second harmonic is the basic restraining criterion for stabilizing differential relays during inrush conditions, certain difficulties arise when protecting such modern transformers [3,7,8].

It is also known that when the inrush current assumes large values, the amount of the second harmonic decreases [4,5].

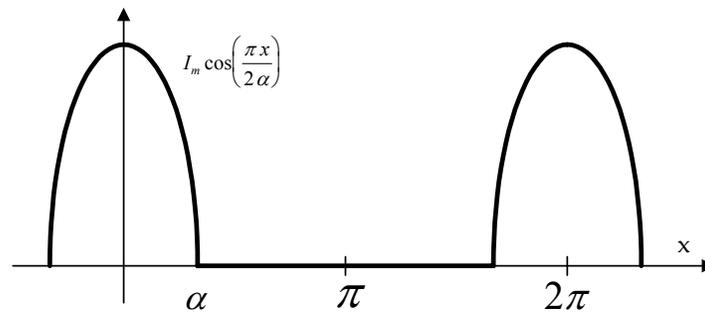


Figure 3. Idealized inrush current for the spectral analysis.

### **2.3. Inrush in three phase transformers**

Inrush currents measured in separate phases of a three-phase transformer may differ considerably because of the following:

- The angle of the energizing voltages are different in different phases.
- When the delta-connected winding is switched-in, the line voltages are applied as the magnetizing voltages.
- In the later case, the line current in a given phase is a vector sum of two winding currents.
- Depending on the core type and other conditions, only some of the core legs may get saturated.

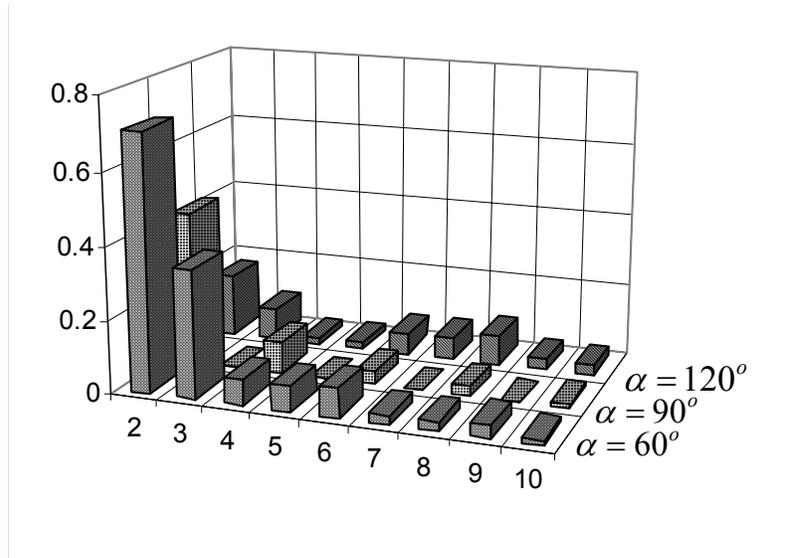


Figure 4. Harmonic content of the idealized inrush current for  $\alpha = 60, 90$  and  $120$  degrees.

As a result of the aforementioned, the current in a particular phase and in a grounded neutral may be either similar to the single-phase inrush pattern (Figure 2) or become a distorted but oscillatory waveform. In the later case, the amount of the second harmonic may drop dramatically, creating problems for differential relaying. Figure 5 presents an example of energizing a three-phase transformer. The currents in the phases A and B assume the typical inrush shape, while the phase C current is an oscillatory waveform.

#### **2.4. Saturation of current transformers during inrush**

Due to the large and slowly-decaying dc component, the inrush current is likely to saturate the CTs even if the magnitude of the current is comparatively low. When saturated, a CT introduces certain distortions to its secondary current (see Figure 6). Due to CT saturation during inrush conditions, the amount of the second harmonic may drop considerably [9].

#### **2.5. Inrush during removal of a fault**

When a near external fault is cleared by an appropriate relay and an associated Circuit Breaker (CB), the voltage at the terminals of a transformer recovers to its normal level. This creates conditions similar to energizing of a transformer, and inrush current may occur. However, two factors make the situation different:

- The step change of the voltage is usually much lower than during switching the transformer in. Only when a three phase solid fault at the interconnected busbar occurs and gets removed, the situation corresponds to switching in.

- Usually, there is no significant offset in the flux generated during an external fault, and therefore, the probability of severe saturation of the transformer core becomes low.

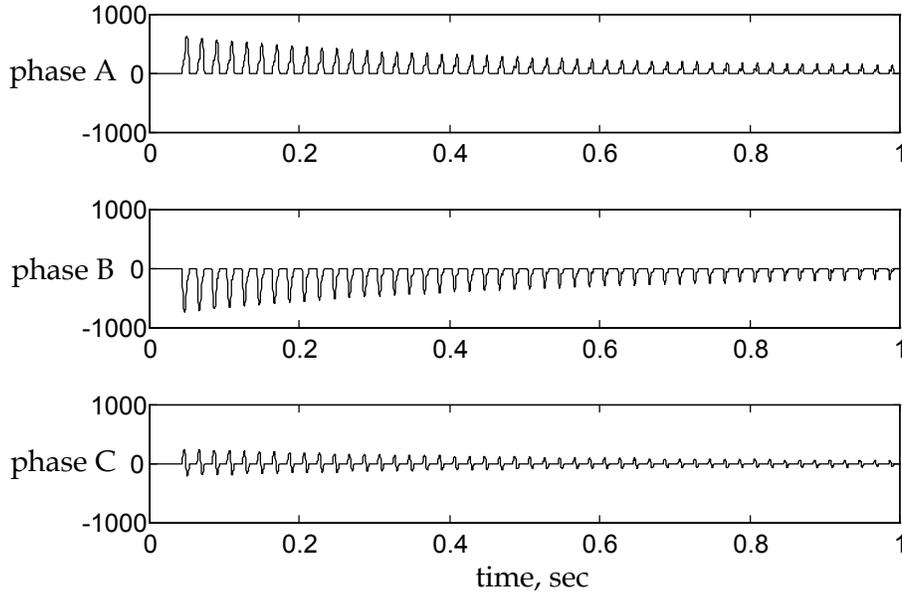


Figure 5. Sample inrush currents in a three-phase wye-delta connected transformer (energizing from the wye side).

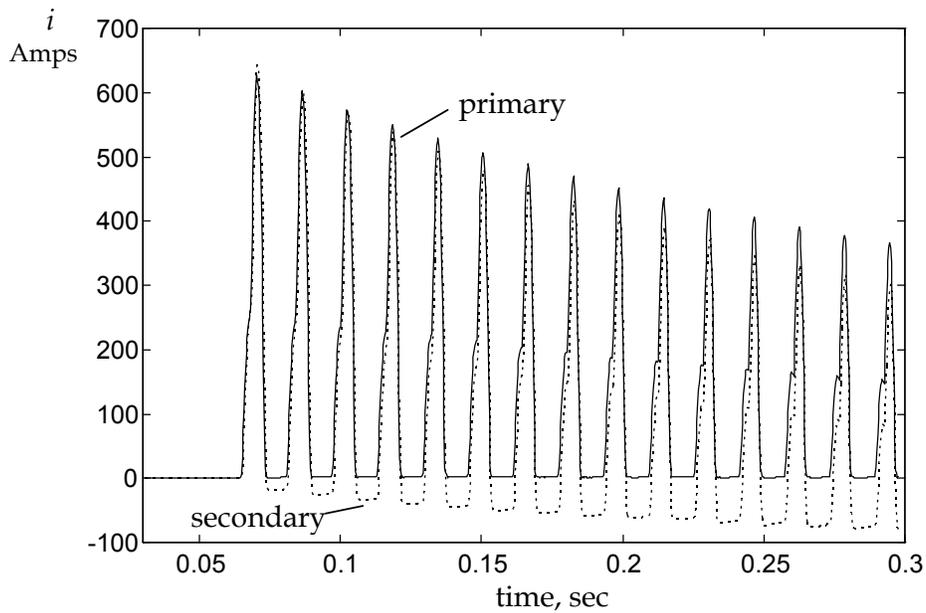


Figure 6. Primary and re-scaled secondary currents during sample inrush conditions under saturation of the CT.

Consequently, the magnitude of the recovery inrush current is significantly lower than in the case of the initial inrush. The shape and harmonic profile of the recovery inrush current are similar to those measured during initial energizing.

### **2.6. Sympathetic inrush**

This phenomenon occurs when a transformer parallel to another, already energized transformer is being energized as shown in Figure 7. Assume, the transformer  $T_2$  has a large positive remanent flux and is switched-in at the unfavorable voltage phase, and obviously, a large inrush current will be drawn by this transformer. The slowly decaying dc component of the inrush current produces a significant voltage drop across the resistance of the equivalent power system (the reactance does not contribute to the voltage drop because the time derivative of the decaying dc component is low). The resulting dc voltage drop shifts abruptly the voltage at the busbar B. The change of the busbar B voltage decreases saturation of the transformer  $T_2$ , and consequently, reduces the inrush current of  $T_2$ . The transformer  $T_1$ , in turn, is exposed to this abrupt change of the voltage and may generate its own inrush current but in opposite direction (Figure 8). The dynamics of the phenomenon is as follows: initially only  $T_2$  draws an inrush current; then  $T_1$  increases its own inrush current while  $T_2$  decreases its current; finally both the currents decay as both the units get completely energized (Figure 8). Because the dc offset of the current in the supplying line is reduced, the damping of this current is also reduced. Consequently, the sympathetic inrush may last much longer as compared to their individual switching-in (even for minutes [4,5]).

Two problems may potentially occur during sympathetic inrush:

- The inrush current in the already energized unit ( $T_1$ ) may be significant enough to cause problems for the protection of this transformer.

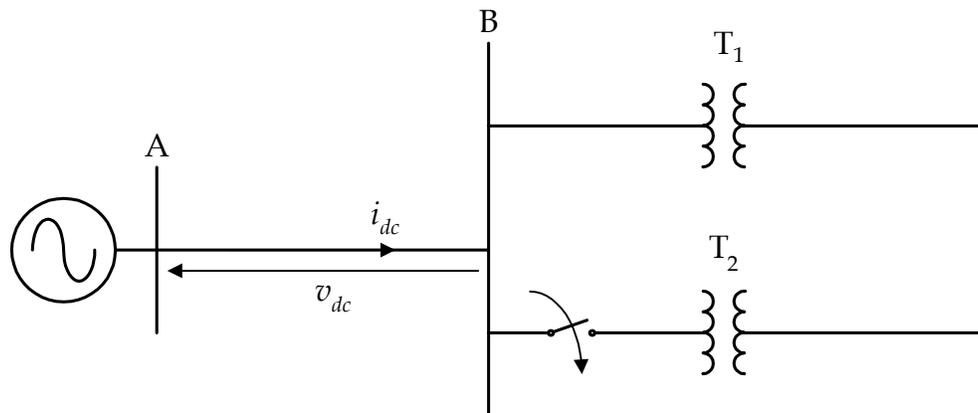


Figure 7. Conditions leading to the sympathetic inrush.

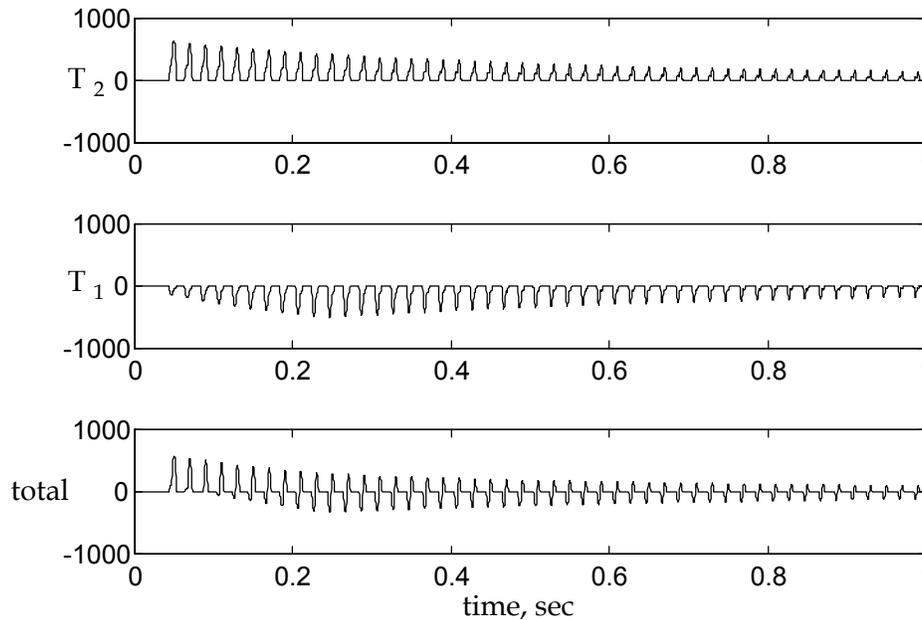


Figure 8. Sample sympathetic inrush currents.

- The current in the supplying line is a vector sum of both the inrush currents, and as such may be similar to an offset fault current. This, in turn, would create problems when the parallel transformers share a common protection system.

### **3. Inrush Restraint Algorithms – A Brief Review**

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Historically, a delay achieved by different means was used to prevent false tripping during inrush conditions. Either the relay was disabled for a given time when switching a protected transformer in, or a special was used [6]. The delay, however, is no longer considered an acceptable means of restraining the differential relay during magnetizing inrush, especially for large power transformers. Modern means of restraining differential relays during magnetizing inrush are by recognizing inrush from the wave shape of a differential current either indirectly (harmonic analysis) or directly (waveform analysis) [8,9,10].

#### **3.1. Harmonic restraint**

This is a classical way to restrain the relay from tripping during magnetizing inrush conditions. As analyzed in section 2, the magnetizing inrush current appearing to a relay as the differential signal, displays certain amounts of higher harmonics. Generally, low levels of harmonics enable tripping, while high levels indicate inrush and restrain the relay. For digital relays this may be written as:

$$TP = I_{CH} < \Delta I_{CD} \quad (2)$$

where:

- $TP$     Tripping Permission from the inrush detector,  
 $I_{CH}$     Combined Harmonic component in the differential current,  
 $I_{CD}$     Combined Differential current,  
 $\Delta$        a threshold.

The condition (2) originates a whole family of algorithms using a variety of approaches in combining currents  $I_{CH}$  and  $I_{CD}$ .

In the simplest approach, the amplitude of the second harmonic in the differential current in a given phase is the combined harmonic signal, while the amplitude of the fundamental frequency component in the differential current in the same phase is used as the combined differential current:

$$\left. \begin{aligned} I_{CH} &= I_{D2 \text{ phase}} \\ I_{CD} &= I_{D1 \text{ phase}} \end{aligned} \right\} \quad (3)$$

Another approach is to use the RMS value for the combined differential current:

$$I_{CD} = I_{D \text{ RMS phase}} \quad (4)$$

When using either form (3) or (4), condition (2) is checked in each phase separately. Extra logic is needed to decide whether or not the entire three-phase relay should get restrained if either one, two or three phases detect inrush conditions.

The relay behavior under such circumstances may be flexibly shaped by using cross-polarization or a cumulative (three-phase) second harmonic.

It is experienced, that the three phase harmonic restraint is more secure [3]. The cumulative restraint defines the combined currents in (2) as sums of the appropriate quantities over three phases:

$$I_{CH} = \sum_{A,B,C} I_{CH \text{ phase}} \quad (5)$$

and

$$I_{CD} = \sum_{A,B,C} I_{D1 \text{ phase}} \quad \text{or} \quad I_{CD} = \sum_{A,B,C} I_{D \text{ RMS phase}} \quad (6)$$

Also, instead of the real RMS, only low order harmonics can be used. In such an approach, the combined differential signal is composed as:

$$I_{CD} = \sqrt{\sum_{k=1}^p I_{Dk}^2} \quad (7)$$

where  $p$  is the highest harmonic measured (usually the fifth harmonic used for restraining the relay during stationary overexcitation conditions).

Depending on the exact formula employed for the combined harmonic and differential signals, the setting  $\Delta$  in (2) assumes slightly different values. Generally, however, the parameter  $\Delta$  is set at about 0.15-0.20 (15-20%).

The harmonic restraint in general, regardless of the method of composing the combined harmonic and differential signals, displays certain limitations.

First, the estimator of the harmonic component (usually the second harmonic only) needs a certain amount of time for accurate estimation of the amplitude. Even if the harmonic is not present in the differential signal at all, the ratio of  $I_{CH}$  to  $I_{CD}$  (2) is initially significantly overestimated (until the fault data fills out the estimator data window). This means that the harmonic restraint usually will not permit tripping for the time approximately equal to the data window length of the estimators (typically one cycle).

Second, in modern transformers the amount of higher harmonics in the magnetizing current may drop well below 10% (the second harmonic as low as 7%, while the total harmonic content at a level of 7.5% [4,5]). Under such circumstances, the setting  $\Delta$  in (2) should be adjusted below 7%. This may lead, however, to delayed or even missing operations of the relay due to the harmonics in the differential currents during internal faults accompanied by saturation of the CTs. Cross-restraint or time-controlled threshold provide only a partial solution to this problem.

Third, the second harmonic ratio may temporarily (for several cycles) drop below the safe 20% due to transients as shown in Section 4.

### **3.2. Waveform-based restraints**

There are basically two inrush restraining methods of this kind [9]:

- the first, and more common approach, pays attention to the periods of low and flat values in the inrush current (“dwell-time” – criterion 1),
- the second algorithm pays attention to the sign of the peak values and the decaying rate of the inrush current (criterion 2).

#### *A. Criterion 1*

The hypothesis of magnetizing inrush may be ruled out if the differential current does not show in its every cycle a period lasting no less than 1/4 of a cycle in which the shape of the waveform is both flat and close to zero (see Figure 9). This relaying principle was

known in the era of static relays and there are certain analog schemes developed for implementing it. Figure 10 shows the common one [6,7].

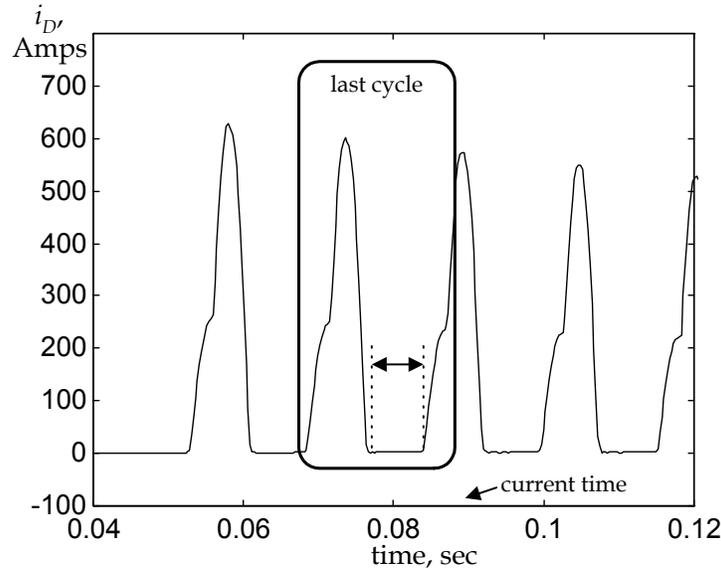


Figure 9. Illustration of the direct waveform recognition of inrush (criterion 1).



Figure 10. Sample analog scheme for direct waveform recognition of inrush (criterion 1).

This form of direct waveform restraining regardless of its implementation shows weaknesses:

- (a) the recognition of an internal fault versus magnetizing inrush takes one full cycle,
- (b) the CTs, when saturated during inrush conditions (very likely due to the dc component in the current), change the shape of the waveform within the dwell periods (Figure 6) and may cause a false tripping,
- (c) during severe internal faults, when the CTs saturate, their secondary currents may also show periods of low and flat values exposing the relay to missing operations.

#### B. Criterion 2

The hypothesis of magnetizing inrush may be ruled out if the differential current [10] (see Figure 11):

- has its peaks displaced by half a cycle, and
- any two consecutive peaks are not of the same polarity.

This method needs robust detection of the peak values. Timing between two consecutive peaks must be checked with some tolerance margin accounting for the frequency deviations.

Theoretically, this method needs three quarters of a cycle to distinguish between internal faults and inrush conditions. The first peak of the fault current appears after a quarter of a cycle, the next one - half a cycle later. With the second peak arriving, the criterion rejects the inrush hypothesis and sets the tripping permit.

As its advantage, this method tolerates deep saturation of the CTs during both inrush conditions and internal faults.

The main disadvantage of this algorithm is the need of cross polarization between the phases. Not always all three phases show the typical inrush uni-polar waveform. Also, during very smooth energization of a protected transformer (what may accidentally happen owing to the adequate relation between the switching angle and the remanent flux), this criterion will fail.

This criterion may be also used in its indirect form as a modifier for the instantaneous differential overcurrent element. Defining the overcurrent principle as:

$$TRIP = |i_D| > \Delta \quad (8)$$

and specifying one threshold, one needs to adjust this threshold very high to prevent false trippings (above the highest inrush current). One may, however, re-define the operating principle (Figure 12):

$$TRIP = (i_D > \Delta_+) \text{ and } (i_D < \Delta_-) \quad (9)$$

and use two thresholds to detect the uni-polarity/bi-polarity of the signal (Figure 12). When using the modified overcurrent principle, the setting may be adjusted as low as one third of the traditional threshold. This allows much more internal faults to be quickly detected by the unrestrained overcurrent algorithm.

### **3.3. Other approaches**

#### *A. Model methods*

This family of approaches solves on-line a mathematical model of a fault-free transformer [11,12]. Either certain parameters of the model are computed assuming the measured signals; or certain fraction of the terminal variables are computed based on all the remaining signals, and next compared to their measured counterparts. In the first case, the values of the calculated parameters differentiate internal faults from other disturbances (including inrush conditions). In the second case, the difference between the calculated

and measured signals enables the relay to perform the classification. These approaches call for voltages and currents at all the terminals to be measured.

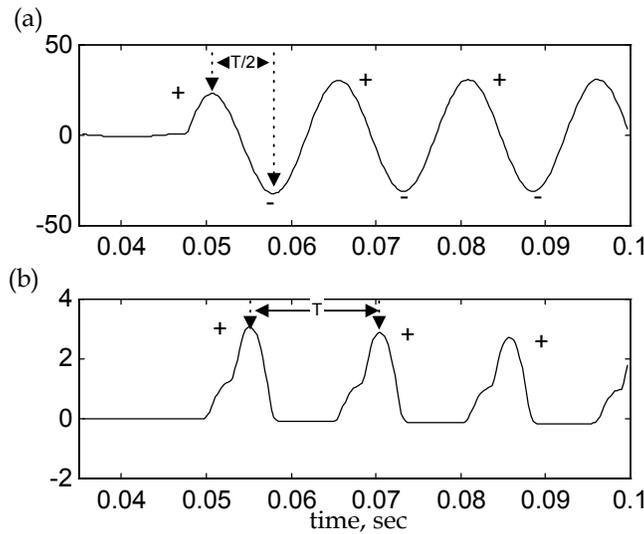


Figure 11. Illustration of the criterion 2. Internal fault (a) and magnetizing inrush (b) currents.

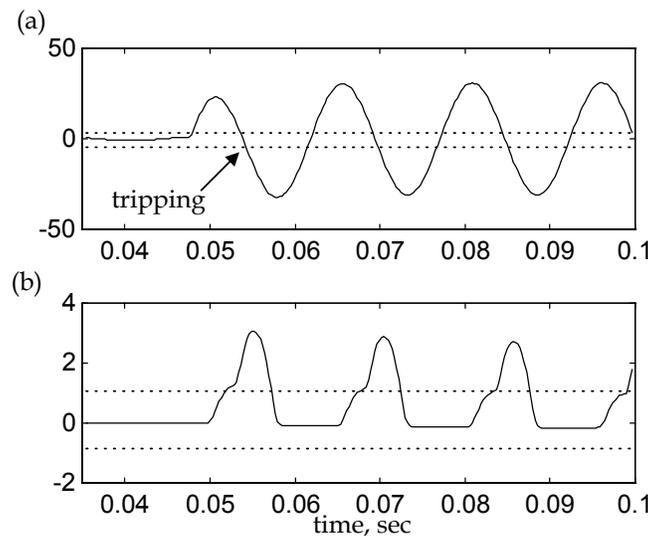


Figure 12. Illustration of the double-threshold overcurrent principle. Internal fault (a) and magnetizing inrush (b) currents.

### B. Differential power method

Another relaying principle uses the differential active power to discriminate between internal faults and other conditions (including magnetizing inrush). Instead of the differential currents, the differential power is computed and monitored [13]. The operating signal is a difference between the instantaneous powers at all the transformer's terminals. This approach calls for measuring the voltages at all the terminals, but pays back by avoiding the vector group (angular displacement between the current and voltages at dif-

ferent windings) and ratio compensation. The dependability of this method may be further enhanced by compensating for the internal active power losses — both in copper, and in iron.

### *C. Flux-based inrush restraint*

This relaying algorithm differentiates internal faults from the inrush and overexcitation conditions based on the on-line calculated flux in the core [14,15]. As its advantage, this approach ties together the cause of the problem (saturation of the core as a source of the current unbalance) with the phenomenon used for recognition (flux in the core).

## **4. A New Algorithm**

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The algorithm presented in this paper deals with the phenomenon of the second harmonic dropping temporarily below the 15-20% level during inrush conditions.

The situation is caused by the large values of the magnetizing current forcing the angle  $\alpha$  in the theoretical considerations (equation (1), Figure 3) to be more than 90 degrees. This in turn results in the second harmonic content to drop below 20% (see Figure 4).

Figure 13 illustrates this phenomenon in the time domain by showing a sample magnetizing current (a) and its second harmonic ratio (b) estimated with the use of the full-cycle Fourier algorithm working at 64 samples per cycle (s/c). In this example, the second harmonic ratio drops below 20% for more than 5 cycles. This, in turn, would either cause a false trip or force the user to set the threshold below 5% jeopardizing both speed and sensitivity of the relay.

As the time passes by, the current resembles more the typical inrush waveform (the parameter  $\alpha$  in (1) decreases) and the second harmonic ratio increases above the safe 20% level.

### **4.1. Algorithm derivation**

The classical second harmonic restraint compares the magnitude of the second harmonic with the magnitude of the fundamental frequency component (or some derivatives of those as explained in subsection 3.1). By following this traditional approach one neglects the other dimension of the derived ratio — the phase relation.

It is obvious that the second harmonic rotates twice as fast as the fundamental frequency phasor. This obstacle is, however, easy to overcome.

Thus, the question has been asked: Can the angle between the second and first harmonics of the magnetizing current, in addition to the amplitude ratio alone, provide better recognition between magnetizing inrush currents and internal fault currents?

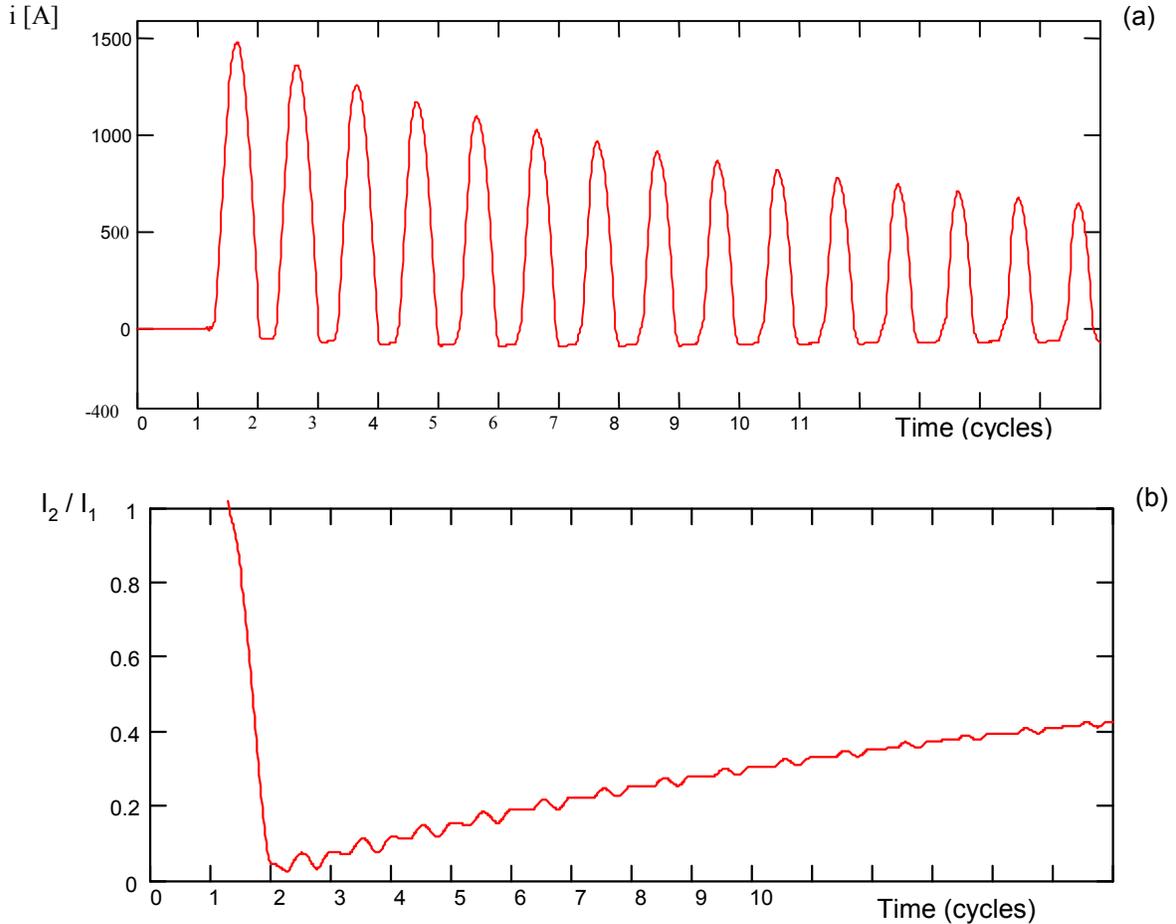


Figure 13. Sample inrush current (a) and its second harmonic ratio (b).

Seeking the answer the following decision (discriminating) signal has been adopted:

$$\underline{I}_{21} = \frac{\underline{I}_2}{\underline{I}_1 \cdot e^{j\omega t}} = \frac{I_2}{I_1} \langle \arg(\underline{I}_2) - 2 \cdot \arg(\underline{I}_1) \rangle \quad (10)$$

where:  $\underline{I}_2$  second harmonic phasor rotating at  $2\omega$  ( $\omega$  - system radian frequency),  
 $\underline{I}_1$  first harmonic phasor rotating at  $\omega$ .

The quantity (10) is referenced with respect to the angle and angular velocity of the second harmonic (by subtracting the phase of the first harmonic multiplied by the factor of 2). In the steady-state, both the magnitude and the argument of the complex number  $\underline{I}_{21}$  are constant.

The traditional second harmonic restraint uses the magnitude of  $I_{21}$  neglecting the phase of it.

## **4.2. Evaluation**

### *A. Analytical evaluation*

In order to evaluate the recognition power of the quantity  $I_{21}$  the simplified inrush current model shown in Figure 3 has been assumed and both the amplitude ratio and phase angle difference between the second and first harmonics have been derived analytically (similarly to equation (1)). It was concluded that the angle assumes the value very close to either +90 or -90 degrees.

The calculations have been repeated for the waveform model that included a decaying dc component of its time constant varied within a wide range. Again, we obtained the analytical proof that the phase angle difference between the second and first harmonics is close to 90 degrees regardless of the amplitude ratio dropping below 20%.

To illustrate this, Figure 14 shows a trajectory of the quantity  $I_{21}$  (a time series of points resulting from the data window sliding along the current waveform shown in Figure 13). As one can see, even though the second harmonic ratio drops almost to zero, the trajectory progresses along the -90-degree line.

It is worth noticing that on the complex plane of  $I_{21}$ , the traditional second harmonic operating region is a circle with the radius of 0.15-0.20. As a result, the traditional relay would falsely trip for this case.

### *B. Statistical evaluation*

The algorithm has been tested using numerous waveforms obtained by simulation and from recordings on physical made-to-scale transformers.

The following factors ensure diversity of the considered cases:

- both wye-delta and wye-wye connections have been taken into account,
- energization from both wye and delta windings have been considered,
- energization onto an internal fault has been considered,
- various inrush factors have been taken into account (weak and strong energizing systems, random residual magnetism, random point-on-wave when energizing, etc.).

The performed analysis has showed improved discrimination ability of the new algorithm comparing with the traditional second harmonic restraint.

To illustrate this, Figure 15 shows a histogram of the new decision signal for numerous inrush cases for energizing from both wye and delta windings. As seen from the figure, the values of the complex second harmonic ratio cluster along the  $\pm 90$ -degree lines.

For comparison, Figures 16 and 17 present histogram of the complex second harmonic ratio for internal faults in a wye-wye and delta-wye transformers, respectively. The new restraint quantity converges at the origin. The values away from the origin are caused by transients and are distributed quite uniformly. Thus, there is insignificant overlapping between the internal fault (Figures 16 and 17) and inrush (Figure 15) patterns. This ensures robust operation of the new algorithm.

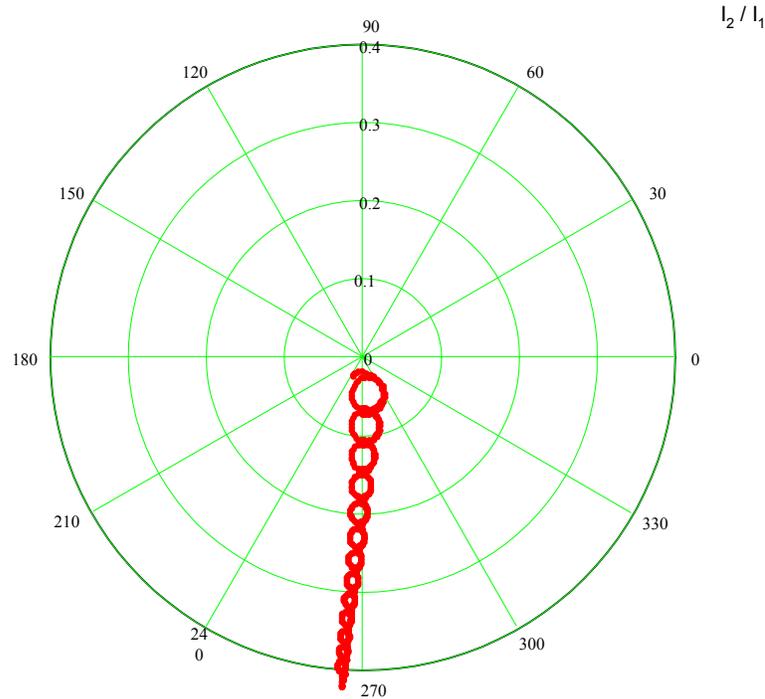


Figure 14. Trajectory of the new decision quantity  $I_{21}$  during a sample inrush wave of Figure 13.

### **4.3. Operate/Restraint regions**

Taking the above considerations into account the operating region for the new decision quantity  $I_{21}$  is shaped as shown in Figure 18.

The following applies to the operate/restraint regions:

- the operating region stretches between approximately  $\pm 20\%$  for angles close to 0 and 180 degrees (traditional second harmonic restraint),
- for angles close to  $\pm 90$  degrees the operating region is cut with two lens-like shapes ensuring blocking operation for low values of the second harmonic,
- the lens-like cut-offs are not stationary, but are made functions of time — initially, the cut-offs are very deep (Figure 18), but after several cycles they disappear leaving a classical circular-like operating characteristic (Figure 19).

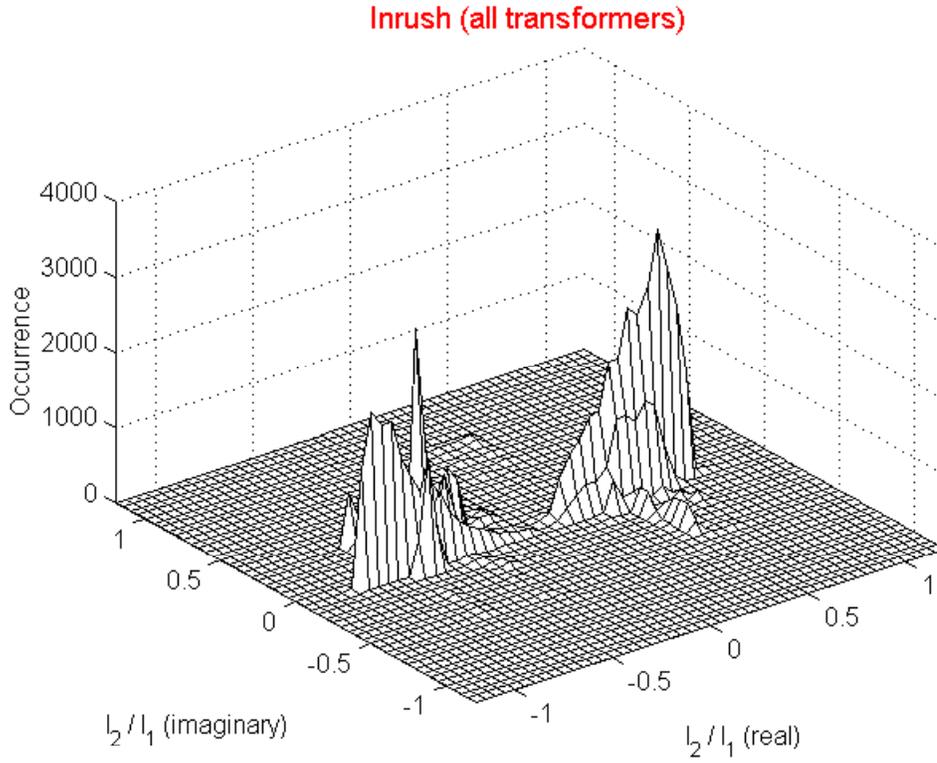


Figure 15. Histogram  $I_{21}$  for numerous inrush waveforms (various transformers).

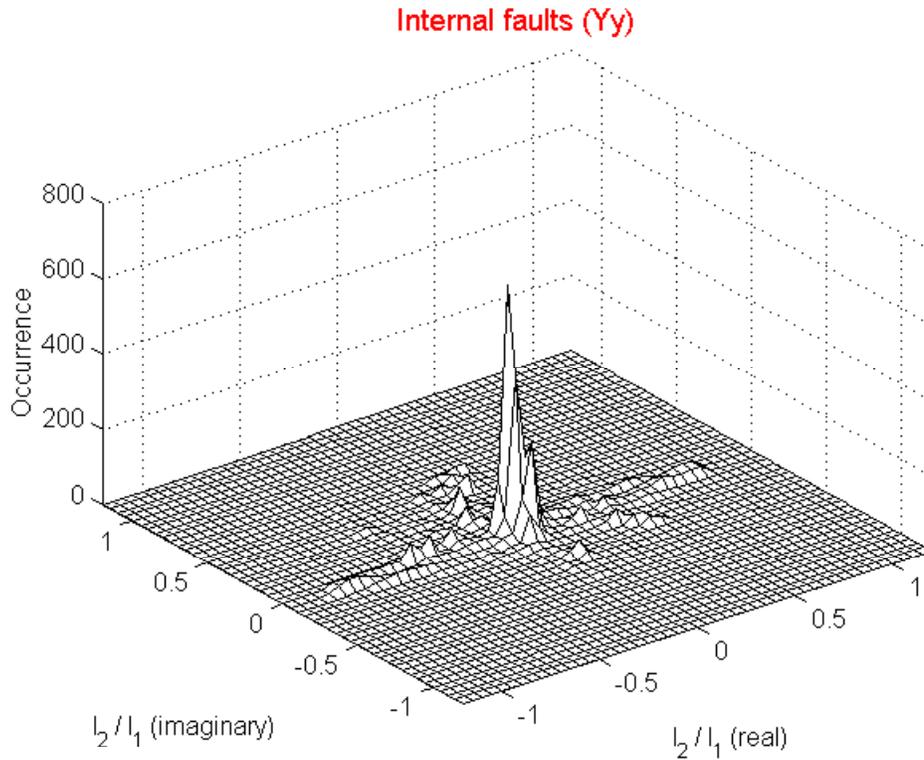


Figure 16. Histogram of  $I_{21}$  for numerous internal fault waveforms (wye-wye transformers).

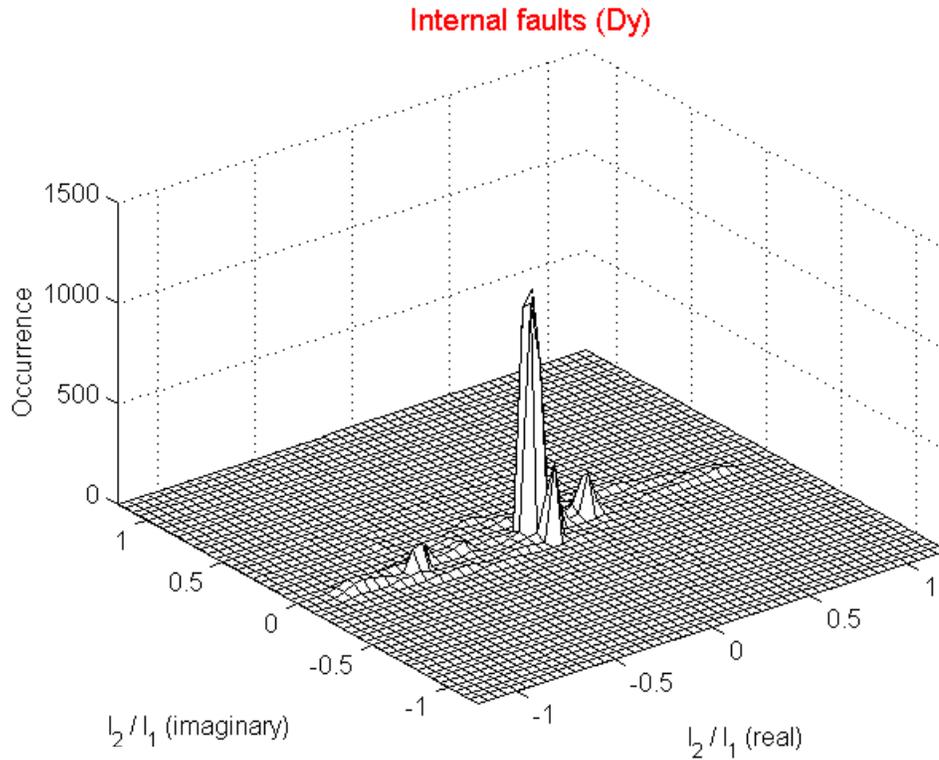


Figure 17. Histogram of  $\underline{I}_{21}$  for numerous internal fault waveforms (delta-wye transformers).

As a result of the dynamic restraint, one obtains a time-dependent operating characteristic for the complex second harmonic ratio. The time required to unblock the relay (i.e. the time after which the magnetizing inrush restraint is taken out) is a function of  $\underline{I}_{21}$ . If the latter does not change in time, the stationary  $t-\underline{I}_{21}$  relation may be derived as shown in Figure 20. The obtained characteristic has the following distinctive features:

- if the angle of  $\underline{I}_{21}$  is close to 0 or 180 degrees, the inrush restraint is removed immediately regardless of the magnitude of the second harmonic,
- if the angle is close to  $\pm 90$  degrees the delay before removing the restraint depends on the amount of the second harmonic: for low ratios of the second harmonic, the delay is very short; while for ratios close to 20% it rises to 5-6 cycles; this is enough to prevent maloperation due to the second harmonic dropping below some 20% during inrush conditions.

#### **4.4. Implementation**

The described algorithm has been implemented using the concept of a “universal relay” — a modular, scaleable and upgradable engine for protective relaying [16]. Figure 21 presents the basic hardware modules of the relay; while Figure 22 — the actual implementation.

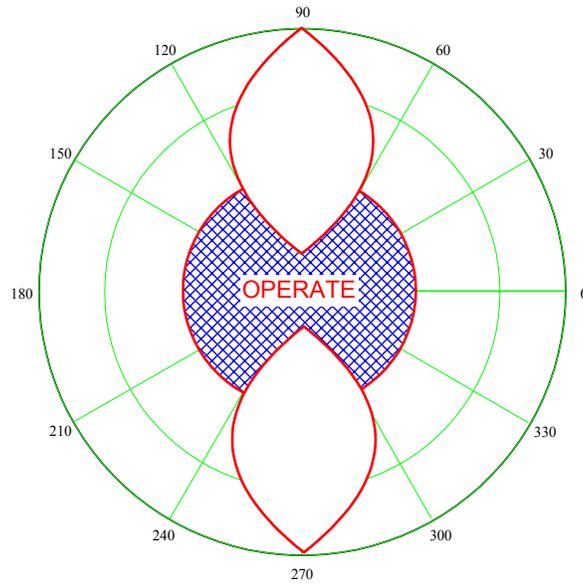


Figure 18. Operating region for the new decision signal.

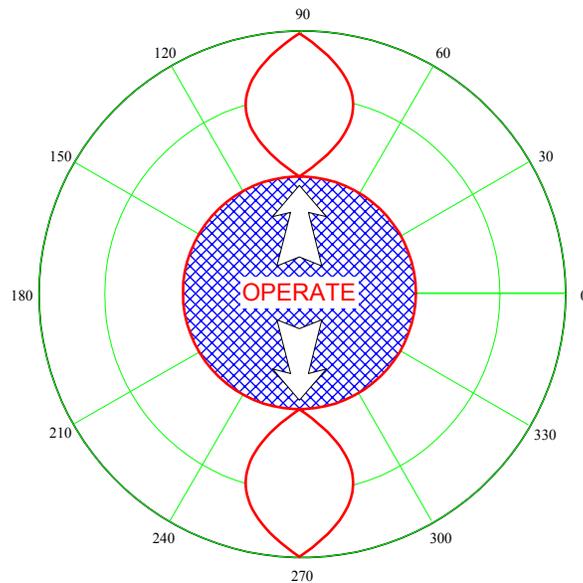
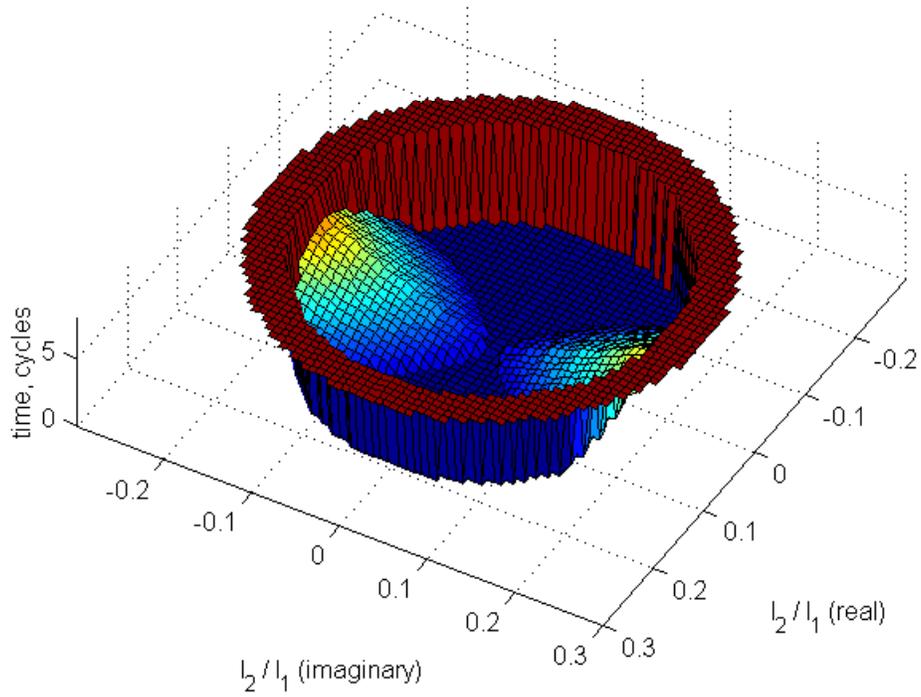


Figure 19. Dynamic expansion of the operate region for the new decision signal.



Isochrone contours, cycles

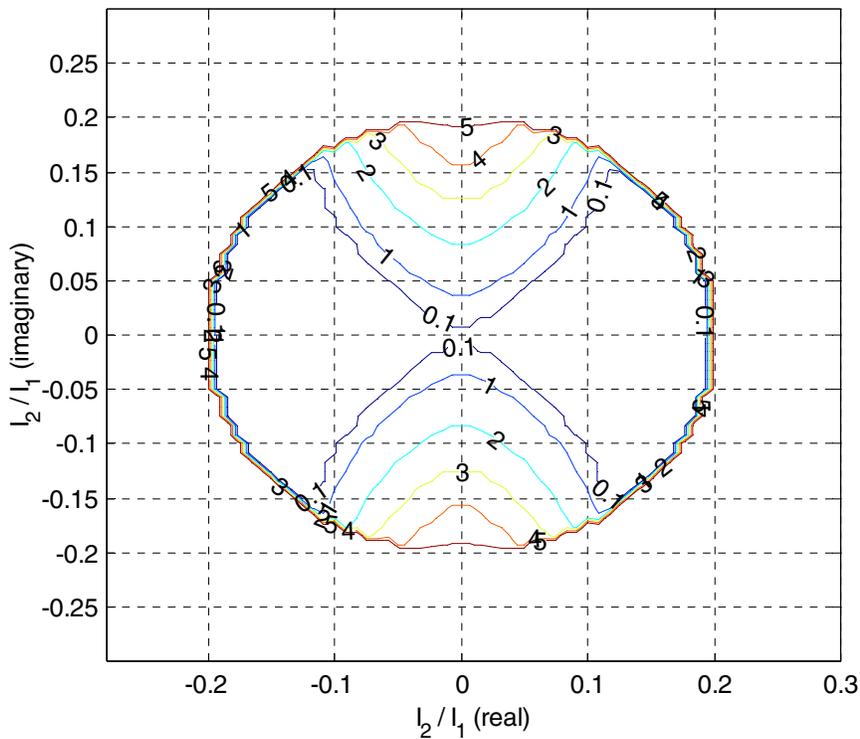


Figure 20. Effective operating characteristic ( $t-I_{21}$ ) for the complex second harmonic restraint (3-D plot and isochrone contours).

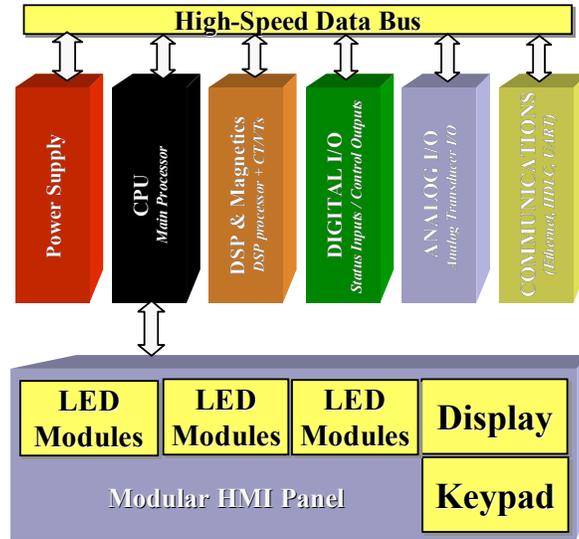


Figure 21. Modular hardware architecture.

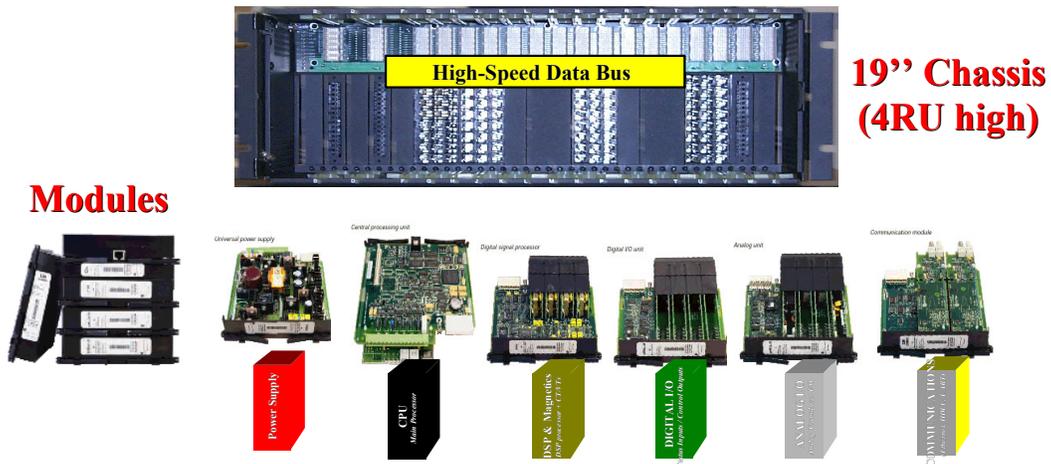


Figure 22. Actual relay architecture.

#### 4.5. Testing

The presented algorithm has been tested using the Real Time Digital Simulator (RTDS [17], Figure 23). Both simulated and field recorded waveforms have been used.

The testing proves a very good performance of the new algorithm.

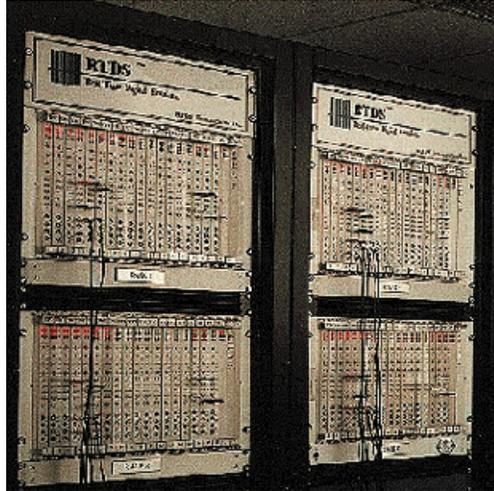


Figure 23. RTDS hardware used in testing.

## 5. Conclusions

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This paper presents a new inrush restraint algorithm for protection of power transformers. The algorithm is an extension of the traditional second harmonic method — instead of measuring the ratio between the magnitudes of the second harmonic and the fundamental frequency component, the algorithm considers a ratio between the phasors of the second and the fundamental frequency components of the differential signal.

The new decision signal has been proposed together with the appropriate operating region. The operating region is made dynamic in order to maximize the relay performance on internal faults.

The new algorithm has been successfully implemented using the universal relay platform.

The results of extensive testing prove that the algorithm enhances the relay stability during magnetizing inrush conditions maintaining - at the same time - the performance on internal faults.

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## **Biographies**

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**Ara Kulidjian** received the B.A.Sc. degree in electrical engineering from the University of Toronto, Canada, in 1991. He then joined GE Power Management where he has been involved in system and algorithm design of protection and control systems. He is a Registered Professional Engineer in the Province of Ontario and a member of the Signal Processing Society of the IEEE.

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